Can Mathematics Map the Way Toward Less-Bizarre Elections?

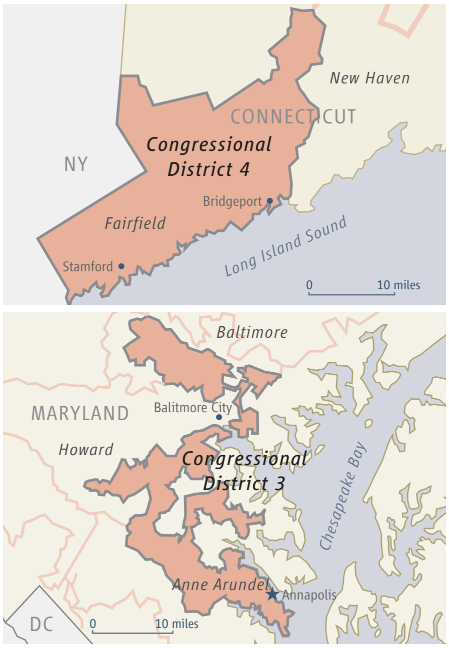
With the 2010 census looming, U.S. politicians and their legal teams are gearing up for another round of wrangling over the spoils of redistricting: the process of deciding which voters get to reelect which members of the House of Representatives and other legislative bodies. Parties in power like to carve up voters to their own advantage, a practice known as gerrymandering. Some reformers, however, hope to limit the mischief—and are turning to mathematics for tools to do so. In a marathon 6-hour session at the Joint Meetings, speakers discussed ideas ranging from pie-in-the-sky theoretical to crust-on-the-ground practical.

The term "gerrymandering" dates back to 1812, when Massachusetts Governor Elbridge Gerry signed into law a tortuous districting map that favored his Democratic-Republican Party over the rival Federalists. But given the fine-grain demographic detail of modern political databases, "the problem is much worse than it used to be," says Richard Pildes, an expert on election law at the New York University School of Law in New York City. Gerrymandering "gives people the sense that they're not really in control of their democracy," Pildes says. "It's part of what contributes to an alienation and cynicism about democracy."

The mathematics of redistricting starts with arithmetic and geometry. Ideally, every district in a state would have an equal population and would be, in some sense, both "contiguous" and "compact." Socioeconomic, political, and racial demographics also come into play. "You can have equipopulous districts and still have whoppingly biased gerrymanders," notes Sam Hirsch, a lawyer at Jenner & Block in Washington, D.C., who specializes in election law and voting rights.

To a mathematician, contiguous means connected—i.e., you can travel from any point in it to any other without leaving the region. Compactness is trickier. Various definitions have been proposed, including one presented at the session by Alan Miller, a graduate student in social science at the California Institute of Technology (Caltech) in Pasadena, California.

Miller's method, developed with Caltech economist Christopher Chambers, quantifies the "bizarreness" of geometric shapes. (The word "bizarre" traces to a 1993 ruling in which the U.S. Supreme Court struck down several oddly shaped congressional districts. Politicians' attempts to handpick their constituents invariably create convolutions in district lines.) In essence, bizarreness is the probability that the most direct path between two randomly chosen voters within a district crosses district lines. The higher the probability, the more bizarre the district is. (The path is required to stay within the state, to avoid penalizing districts that sit on ragged state boundaries.)



How bizarre. Researchers can rank the shapes of Congressional districts ranging from highly compact (top) to convoluted.

Using block data from the 2000 census, Miller and Chambers have computed bizarreness for the congressional districts of Connecticut, Maryland, and New Hampshire. Most compact was Connecticut's 4th District, with bizarreness 0.023; most oddly shaped: Maryland's 3rd district, at 0.860 (see figure).

Bizarreness could be used as a threshold criterion in producing redistricting maps or comparing alternatives, Miller says. "You can use it to reject districts that are badly shaped."

In his own proposal, Hirsch took the idea of thresholds and added a dose of high-octane competition. Rival factions—or anyone else interested in entering the fray—would be able to counter one another's maps, as long as each new submission improved on at least one of three criteria and matched the other two. The goals of the three criteria are to minimize the number of counties cut up by district lines, equalize as much as possible the number of districts leaning toward each of the two major parties, and maximize the number of "competitive" districts, in which neither major-party candidate in a recent statewide contest would have won by more than 7% of the vote.

Hirsch's proposal "is a great idea," says Charles Hampton, a mathematician at the College of Wooster in Ohio, who has been involved in redistricting since the early 1980s. (He drew maps in 1991 for the governor of California's Independent Redistricting Panel.) "We quibble on some of the details," Hampton says, but "I think [it] has some real prospect of producing a much better situation."

No one expects mathematics to solve the problem to everyone's satisfaction. "It's ultimately a political problem," Hirsch says. Kimball Brace, head of Election Data Services in Manassas, Virginia, and a member of the 2010 Census Advisory Committee, agrees. "Redistricting is contradictions out the wazoo," Brace says. "One person's equality is another person's gerrymander." Nonetheless, a growing group of practitioners believe mathematics can play a key role. Says Pildes, "Math can give you tools for creating processes that are likely to lead people to feel that the process is fair and that the outcome is therefore something to be respected."

Taking a Cue From Infinite Kinkiness

"Clouds are not spheres, mountains are not cones, coastlines are not circles," fractal pioneer Benoit Mandelbrot famously wrote. Pool tables, on the other hand, *are* polygons. But mathematicians can ask, "What if they weren't?"

Robert Niemeyer, a graduate student at the University of California, Riverside, and his adviser, Michel Lapidus, are exploring the mathematics of fractal billiards, in which a point-mass cue ball rattles around inside a shape whose boundary seemingly

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