INCENTIVES IN EXPERIMENTS: A THEORETICAL ANALYSIS†

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ABSTRACT. Experimental economists frequently pay subjects for multiple independent decisions. With this payment mechanism, choice in one decision may be distorted by the choices made in others. If this occurs, incentive compatibility is said to be violated. Such experiments are incentive compatible only if a 'no complementarities at the top' (NCaT) condition is assumed. Assuming subjects' preferences for gambles respect monotonicity (dominated gambles are never chosen), we show that paying for one random decision—the *Random Decision Selection* (RDS) mechanism—is incentive compatible. If nothing more is assumed, then this is essentially the *only* incentive compatible payment mechanism.

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I: INTRODUCTION

The power of the experimental method lies in the researcher's ability to control the environment. In economics, this control enables us to observe choice in isolation, abstracting away the complexities of the field. When subjects are paid for multiple decisions in a single experiment, however, control may be lost because the expected payment from one decision may impact subjects' choices in another. Whether or not subjects treat each decision as if it were in isolation depends on the incentives in the experiment. If they do not, then the experiment is not incentive compatible, and underlying preferences in each decision are not truthfully revealed to the researcher.

Despite this issue, it is common practice in modern experimental economics to pay for multiple decisions (see Table I below). A proposed alternative is to pay for one randomly-selected decision—what we call the Random Decision Selection (RDS) mechanism. Although this apparently solves the complementarities problem, there are examples of preferences for which the RDS mechanism is not incentive compatible (Holt, 1986; Karni and Safra, 1987). Thus, exact conditions under which this mechanism is incentive compatible are not well understood. Neither is it known whether other mechanisms can be used to guarantee truthful revelation of choices in experiments with multiple decisions.

In this paper, we develop a choice-based theoretical framework for studying incentives in experiments. This is done with absolutely minimal assumptions. We do not require the independence axiom, or even probabilistic sophistication. Subjects need not have correct beliefs about randomizing devices such as bingo cages or die rolls. All that is assumed is that subjects have preferences over the outcomes of the experiment, and that their ranking of gambles over outcomes respects the monotonicity axiom that dominated gambles are never chosen. Under this weak assumption, we verify that the RDS mechanism is in fact incentive compatible, and that incentive compatibility fails if the axiom is weakened substantially. Thus, example preferences where the RDS mechanism is not incentive compatible contain monotonicity violations. More importantly, we show that the RDS mechanism (or, a slightly generalized version of it) is the *only* incentive compatible mechanism when the researcher knows little more about preferences than monotonicity. Thus, experimentalists who use other payment mechanisms run the risk that the data will be distorted by the mechanism's incentives.

Obviously, some researchers are explicitly interested in the interactions across decisions. Throughout our paper, we refer to a decision problem as a set of choices meant to be taken in isolation from all others. If an experimenter is interested in how two choices interact, then we view that pair of decisions as part of the same decision problem. An experiment is said to be *incentive compatible* if the payment mechanism induces the subjects to choose their true favorite element in every decision problem.

The incentive compatibility problem arising from multiple decisions has been known since at least Allais (1953). Grether and Plott (1979) and Cox and Epstein (1989) both point to complementarities across decision problems (specifically, income effects) as a possible explanation for observed preference reversals in past work. Yaari (1965, p.285) is one of the first examples of the RDS mechanism being used in practice, offering it as a solution to the complementarities problem.

After identifying the preferences under which the RDS mechanism is incentive compatible, we characterize the entire set of incentive compatible mechanisms, assuming now that a rich set of preferences are admissible. The set of incentive compatible mechanisms is not much larger than the family of RDS mechanisms; essentially, one must either pay for one randomly-selected decision problem that the subject actually faces, or else a hypothetical decision problem for which the subject's optimal choice can always be inferred from their actual choices. Such hypothetical decisions are called surely identified sets, and we refer to this broader family of mechanisms as Random Set Selection (RSS) mechanisms. To our knowledge, the only example of an RSS mechanism being used that pays based on a hypothetical decision is in Krajbich et al. (2010).[1]

Our framework follows Savage (1954), modeling random gambles as *acts* that map states of the world into outcomes. This allows for unobservable, subjective (and even non-probabilistic) beliefs by subjects. Even when an experimenter draws balls from an urn or rolls a die, a subject who misunderstands the device or perceives a malevolent nature (Ozdenoren and Peck, 2008, e.g.) may have beliefs that differ substantially from the long-run frequency of the randomizing device's outcomes. The Savage framework naturally encompasses such preferences. In a companion paper (Azrieli et al., 2012), we explore incentives in experiments assuming subjects view gambles as lotteries with objective probabilities. Although this adds substantial structure to the problem, the results are qualitatively similar to those generated in the Savage framework.

Despite the fact that incentive compatibility of the RDS mechanism has been proposed (at least informally) for almost 60 years, it is not the standard mechanism in use. A quick survey of experimental papers published in 2011 in both top journals and in

[1]Wang et al. (2010) compare this mechanism to other payment mechanisms and find little difference in resulting preference estimates.

Number of Decisions Chosen for Payment:	None	One (RDS)	Three or Four	All	Other Mechanism	Not Specified	Total
'Top Five' Journals	3	2	0	13	0	3	21
Experimental Economics	2	3	2	14	3	2	26

TABLE I. Payment mechanisms used among experimental papers published in 2011. The 'top five' journals are *Econometrica*, *American Economic Review*, *Journal of Political Economy*, *Quarterly Journal of Economics*, and *The Review of Economic Studies*.

the field journal *Experimental Economics* reveals that only about ten percent of experiments employ the RDS mechanism (Table I). Most experimenters prefer to pay for every decision.

One commonly-stated reason to pay for every decision is that it reduces the variance in subjects' payments. For example, in a single experimental auction, the winning bidder may earn \$20, while all others earn \$0. Subjects face significant risk. Paying instead for twenty 'scaled-down' auctions that pay \$1 to the winner significantly reduces payment variance. Incentive compatibility is at risk, however, because this payment mechanism introduces a clear incentive to engage in riskier behavior through both portfolio effects and income effects. In short, if the decision of interest pays either \$20 or \$0, and a subject can leave the lab with some amount other than \$20 or \$0, then the researcher has studied a different decision.

Another argument for paying every period is that complementarities are a second-order phenomenon, swamped by treatment effects. Thus, there is no need to use the RDS mechanism. This is a plausible argument, so we formalize exactly the condition on preferences necessary to use the pay-for-all mechanism. This condition—called No Complementarities at the Top (NCaT)—says that if any subset of decision problems is chosen and the subject is paid their most-preferred item from each problem, then that bundle is preferred to any other bundle they could receive from the same subset of problems. In other words, the bundle of favorites must also be the favorite bundle. Whether or not such an assumption is appropriate clearly depends on the nature of the decision problems under investigation.

Some researchers have suggested that learning across decisions will cause the RDS mechanism not to be incentive compatible. There is a subtle difference, however, between complementarities and learning or framing effects. Dictator game giving may still be affected by previous dictator game outcomes, even when the RDS mechanism is used. Garvin and Kagel (1994) show that subjects learn to avoid the winner's curse mistake in experimental auctions simply by observing others make the mistake. Weber (2003) even shows evidence of 'non-observational' learning across decision problems when no feedback is given. These do not represent failures of the RDS mechanism; rather, these are examples of preferences themselves being shifted by past experience. Plenty of evidence suggests that learning and framing effects can be of first-order concern to experimentalists. When learning or framing effects occur, the RDS mechanism will still be incentive compatible and will still elicit choice truthfully, but the choice that is elicited will be altered by subjects' exposure to other choices or information. These effects would be present in *any* payment mechanism (as long as the relevant stimulus remains present), and so the RDS suffers equally, but maintains its relative advantage of avoiding complementarities across problems.

Another criticism of the RDS mechanism comes from Holt (1986), Karni and Safra (1987), Cox et al. (2011), Harrison and Swarthout (2011), and others. Assuming subjects view gambles as objective lotteries, these authors show, through examples, that the RDS mechanism may not be incentive compatible when subjects have non-expected utility preferences but reduce compound lotteries to simple lotteries. Our results demonstrate exactly why this occurs: It is well-known that if both the monotonicity axiom and the reduction of compound lotteries are assumed, then the independence axiom must hold. The contrapositive of this statement is that preferences which violate the independence axiom but maintain reduction must violate monotonicity. Monotonicity is crucial for the incentive compatibility of the RDS mechanism, and so it should not be used when subjects are suspected of violating independence while satisfying reduction. Our companion paper (Azrieli et al., 2012) discusses this issue in greater detail.

There have been some attempts to explore the incentive compatibility of the RDS mechanism in the laboratory. For example, Laury (2005) finds that, in the Holt and Laury, 2002 procedure for estimating risk aversion, subjects become apparently more risk-seeking when all lotteries are paid. Thus, complementarities (specifically, portfolio effects) can systematically distort the experimenter's estimates of subjects' preferences.

Experimental results by Wilcox (1993), Moffatt (2005), and Baltussen et al. (2010) suggest that incentive compatibility of the RDS mechanism may fail if decisions are chosen with small probabilities or if the decision problems are sufficiently complex. Our theoretical framework reveals that these simply represent particular ways in which the monotonicity axiom may be violated. If a subject facing 100 complex decision problems makes sub-optimal choices on some of the problems, then, under the RDS mechanism, he is selecting acts (payments) that generate weakly worse outcomes in every state of the world, and strictly worse outcomes in at least one state. Monotonicity is being violated. But it seems equally plausible that other payment mechanisms will suffer in this regard, even if NCaT is assumed. The obvious recommendation here is for experimenters to limit the number and complexity of decisions subjects are asked to make, especially when the overall earnings are kept to a standard $20 or $30 per subject.

Wakker et al. (1994) claim that the 'isolation effect' identified by Kahneman and Tversky (1979) justifies use of the RDS mechanism even when independence fails since, in practice, subjects apparently treat gambles in isolation. Several experimental results provide support for this claim (see Camerer, 1989; Starmer and Sugden, 1991; Cubitt et al., 1998; and Hey and Lee, 2005, for example). Cox et al. (2011), however, find evidence that subjects' choices in a decision problem change when a second problem is added and the RDS mechanism is used. This suggests either that framing effects altered subjects' preferences, or that the mechanism was not incentive compatible due to some violation of monotonicity. In our view, either conclusion is troubling, and more experimental work in this area is warranted.

A related strand of literature studies the implementation of infinitely repeated games in a laboratory setting. We view each repeated game as a single decision problem; thus, our results suggest how to choose which repeated games are chosen for payment, while this complementary literature discusses how to pay *within* a repeated game. The standard methodology there—established by Roth and Murninghan (1978)—is to pay for each period, terminating the repeated game with a fixed probability after each period. Such games are often called 'indefinitely-repeated'. Frechette et al. (2011) test this payment scheme against one in which one randomly-selected stage game is chosen for payment.[2] Similarly, Sherstyuk et al. (2011) compare the payment of all periods, the payment of only the last period, and the payment of one randomly-selected period. Chandrasekhar and Xandri (2011) show theoretically that paying for one randomly-selected stage game alters the repeated game, so that players discount future payments more than if each period were paid. Subjects' behavior in the laboratory responds in exactly this way. Since we view a repeated game as a single decision problem (whose choices are entire repeated-game strategies), our results only suggest that experimenters should pay for one randomly-chosen repeated game; which payment scheme is used *within* a repeated game depends on the structure of the repeated game the researcher intends to study.

[2]Frechette et al. (2011) also test payment mechanisms of Cabral et al. (2011) and Cooper and Kuhn (2010).

Our theoretical contribution in this paper is to introduce appropriate notions of payment mechanisms and incentive compatibility specifically tailored to experimental situations. The notion of incentive compatibility is certainly not new, dating back to the early mechanism design literature (for example, see Vickrey, 1961, Hurwicz, 1972, Green and Laffont, 1977, and Myerson, 1981); our work shares much in spirit with these classical notions, but there are differences. We are not interested in eliciting an entire preference relation (as in the classical mechanism design literature), but rather a finite collection of choices. At the same time, we also desire a *strict* notion of incentive compatibility: It is not enough that a decision maker be weakly willing to announce her 'true' choices, but also that she would never be willing to announce false choices. These mark significant departures from the classical literature on two points:

(1) The exogenously-given experiment effectively defines (and, therefore, restricts) the space of allowable messages. Experimenters typically ask subjects to make a collection of choices. This limits our ability to construct appropriate mechanisms, since any two preference relations making the same choices must be treated identically by our mechanism. Full separation is impossible.
(2) Since experimenters are interested in inverting the choice function to uncover underlying preferences, we are forced to consider a strict version of incentive compatibility, ensuring that false reports are never optimal.

The strict notion of incentive compatibility makes the incentive compatibility problem extremely difficult. As we establish, it is rare that a deterministic mechanism will allow for strict incentive compatibility. This is why we must work with random mechanisms. The papers closest to ours in this respect are Gibbard (1977), Barbera (1977), and Barbera et al. (1998). In fact, the idea of extending a given preference over outcomes to an extension over lotteries that respects monotonicity first appears in these works. Our main characterization result—placed in the domain of objective lotteries—is related to the main result of Gibbard (1977). Gibbard takes interest in a multiple-agent environment where payoffs are probabilistic. He seeks to elicit the entire preference relation, and only requires weak incentive compatibility. Gibbard's characterization is geometric, and he never explores the 'random selection' aspect of the rules. Our goal is to understand incentive compatible rules from an experimental design standpoint, so the practical and intuitive structure of the incentive compatible rules as random selection mechanisms is a crucial point.

On the decision theory side, the assumption that preference extensions satisfy monotonicity is one of the most important and fundamental axioms of decision theory. Machina and Schmeidler (1992) suggest that it is the most meaningful weakening of preference which can capture the idea that preference is based on probabilities, yet not satisfy the independence axiom. Monotonicity is explicitly presented as an assumption in nearly all modern works on decision theory that deal with preferences over acts, and many with lotteries as well. It seems to make its first formal appearance in early works axiomatizing subjective expected utility (Savage, 1954, for example). Anscombe and Aumann (1963) also state it formally, and note that it is much in the same spirit as the substitution axiom of Luce and Raiffa (1989) (which itself is only one part of the independence axiom of decision theory). Works relying on some version of this axiom include Gilboa and Schmeidler (1989); Schmeidler (1989); and Maccheroni et al. (2006) for example.

Regardless, Diamond (1967) demonstrates that monotonicity may not be compelling when ex-ante fairness of a procedure may be of concern.[3] We also describe another condition where monotonicity might naturally fail; where the objects of choice are themselves acts, and subjects may believe there is some correlation between states of the world, and the states corresponding to the mechanism. Nevertheless, most decision theories which do not require monotonicity are either attempts to accommodate the Diamond-Machina example, or are more philosophical exercises, designed to provide the weakest possible notion of preference being based on probabilities.

Finally, part of our work focuses on settings where payment mechanisms induce acts over acts. If subjects view these acts as lotteries, and if they reduce compound lotteries down to simple lotteries, then the monotonicity axiom becomes equivalent to the independence axiom. This is already the driving force behind the Luce and Raiffa (1989) axiomatization of expected utility, for example. Furthermore, the examples given by Holt (1986) and Karni and Safra (1987) that show how the RDS mechanism may not be incentive compatible without the independence axiom are actually examples of monotonicity violations, because the authors assume reduction. Segal (1990) provides a host of interesting axioms and characterization theorems for the lotteries-over-lotteries framework. The crux of his argument is that reduction is far from a necessary axiom of decision theory. Dillenberger (2010) continues in this tradition. See our companion paper (Azrieli et al., 2012) for a further discussion of the lotteries framework.

[3]Diamond actually argues against the independence axiom, though Machina (1989) explains that monotonicity itself is the source of difficulty.

II: The General Framework

The set of possible *choice objects* is given by the set X, which is assumed to be finite.[4] The decision maker (also called the *subject*) has a complete, transitive preference relation $\succeq$ over X. For any $x \in X$, let $L(x,\succeq) = \{y \in X \ : \ x \succeq y\}$ and $U(x,\succeq) = \{y \in X \ : \ y \succeq x\}$ be the lower- and upper-contour sets of x according to $\succeq$, respectively. The $\succeq$-*dominant* elements of any set $E \subseteq X$ are denoted by

$$\text{dom}_{\succeq}(E) = \{x \in E : (\forall y \in E \setminus \{x\}) \ \ x \succeq y\}.$$

Where applicable, strict preference (the asymmetric part of $\succeq$) is denoted by $\succ$.

The researcher has an exogenously-given list of k decision problems, denoted $D = (D_1,\ldots,D_k)$, where each decision problem D_i is a set of choice objects from which the subject is asked to choose.[5] Thus, $D_i \subseteq X$ for each $i \in \{1,\ldots,k\}$. Let $\mathscr{D} = \{D_1,\ldots,D_k\}$ represent the set of decision problems. We assume throughout that each $D_i \in \mathscr{D}$ is non-trivial, meaning $|D_i| > 1$, and that the same decision problem does not appear more than once, meaning $D_i \neq D_j$ whenever $i \neq j$. These assumptions are made only to simplify notation and can easily be relaxed.

The researcher does not know the subject's preference relation $\succeq$, but wants to learn the subject's most-preferred element of each D_i. For example, D_1 may be a set of lotteries used to estimate the subject's degree of risk aversion, and D_2 may be a set of intertemporal choices. A researcher studying correlations between risk aversion and patience may run an experiment using only these two decision problems.

Since choices are restricted to D, the choice data from the experiment can be thought of as an announced choice vector $m = (m_1,\ldots,m_k)$. We sometimes call this the subject's *message*. The space of all possible messages is $M = \times_i D_i$. For each $i \in \{1,\ldots,k\}$, let

$$\mu_i(\succeq) = \text{dom}_{\succeq}(D_i)$$

[4]Most experiments explicitly force choices to be from a finite set or grid. Those that don't can still be thought of as having finite X since there exist limits on precision imposed by computer interfaces and finite languages.

[5]This framework does not require that subjects choose consumption goods directly. They may submit abstract announcements that map into consumption goods. For example, the experimenter could ask the subject to announce their preferences and then reward them a consumption good (such as money) based on their announcement. In that case, X would represent the space of announcements, where each announcement maps to a consumption good payment. These payments may include randomness or uncertainty. As long as underlying preferences for consumption goods extend to the space of announcements, then the preference relation $\succeq$ over X is still well-defined.

be the set of $\succeq$-dominant elements of D_i, and define $\mu(\succeq) = \times_i \mu_i(\succeq)$. We refer to $m \in \mu(\succeq)$ as a *truthful* message for $\succeq$.[6]

The researcher pays the subject—based on their message—using objects from X. For example, if $D_1 = \{\text{cake,pie}\}$ and the subject announces $m_1 = \{\text{cake}\}$, he may be given a piece of cake as payment for that decision. If the subject is paid something not in $\bigcup_i D_i$, then X can be expanded to include the payment objects as well as the choice objects.

A randomizing device may be used to select payment objects from X. We adopt the subjective uncertainty framework of Savage (1954), modeling such randomization as an *act*. An act is a mapping from a state space Ω into X. The space of all acts is X^Ω. We view Ω as capturing all relevant states of the world, with each generically being denoted by ω. Randomizing devices (or, acts) partition Ω into a collection of events, and assign an outcome to each event. For example, a coin flip defines two events $H \subseteq \Omega$ and $T \subseteq \Omega$, with $H \cup T = \Omega$ and $H \cap T = \emptyset$. States $\omega \in H$ are those for which the coin lands 'heads', and states $\omega \in T$ are those for which the coin lands 'tails'. An act f that pays {cake} if heads and {pie} if tails would have $f(\omega) = \{\text{cake}\}$ if $\omega \in H$, and $f(\omega) = \{\text{pie}\}$ if $\omega \in T$.

We assume throughout that Ω is finite.[7] A constant act is one that pays the same object in X regardless of the state. For example, if $f(\omega) = \{\text{cake}\}$ for every $\omega \in \Omega$, then f is a constant act.

In an experiment, payments depend on the subject's announced choices. A *(payment) mechanism* ϕ therefore takes the announced choice m and awards the subject with an act in X^Ω. Thus, $\phi : M \to X^\Omega$. If ϕ is a payment mechanism, then $\phi(m)(\omega)$ identifies the choice object in X that is paid if the subject announces choice vector m and state ω obtains.

We refer to the pair (D,ϕ) as an *experiment*; D completely specifies the choices the subject must face, and ϕ describes how they are paid for those choices. Since D determines the domain of a mechanism, there is little distinguishing an experiment (D,ϕ) from its associated mechanism ϕ; when it causes no confusion, we refer to experiments and mechanisms interchangeably.

The original preference $\succeq$ is defined over elements of X, not X^Ω. But payments are objects in X^Ω, and these payments must be evaluated by the subject when making their

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[6]Completeness of $\succeq$ is not needed for most of our results. Suppose X can be partitioned into mutually-exclusive sets $X_1, X_2, \ldots, X_r$ such that every decision problem D_i is contained completely within one X_j. In that case, $\succeq$ only needs to be complete within each X_j; elements from X_1 need not be comparable to elements from X_2, for example.

[7]Assuming a finite state space is acceptable here since payment mechanisms need to be contractible if they are to be incentive compatible, and so the events on which payments can depend must be describable using finite language.

choices. To this end, we assume that the subject has a complete and transitive preference $\succeq^*$ on the space X^Ω which "extends" $\succeq$ in a sense to be made precise. To avoid confusion, we henceforth refrain from calling $\succeq^*$ a preference relation; instead, we refer to it as an extension of $\succeq$. The asymmetric relation $\succ^*$ denotes the asymmetric part of $\succeq^*$.

We assume very little about structural properties of admissible extensions. In particular, no version of the independence axiom or sure thing principle is assumed. At this point, we assume only that extensions agree with the underlying preferences when restricted to the space of constant acts. Formally, if $f(\omega) = x$ and $g(\omega) = y$ for all ω, then $f \succeq^* g$ if and only if $x \succeq y$. In general, let the correspondence $\mathscr{E}$ define the set of admissible extensions, where $\mathscr{E}(\succeq)$ identifies the set of admissible extensions for the preference relation $\succeq$. All $\mathscr{E}$ must be such that each $\succeq^* \in \mathscr{E}(\succeq)$ agrees with $\succeq$ on the space of constant acts.

A successful experiment is clearly one in which the payment mechanism induces the subject to announce their choices truthfully, regardless of their preferences. We refer to this as *incentive compatibility* of the experiment, and note that whether or not a mechanism (or experiment) is incentive compatible depends crucially $\mathscr{E}$.

Definition 1 (Incentive Compatibility). A mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if, for every preference $\succeq$, every extension $\succeq^* \in \mathscr{E}(\succeq)$, every truthful message $m^* \in \mu(\succeq)$, and every message $m \in M$, we have that $\phi(m^*) \succeq^* \phi(m)$, with $\phi(m^*) \succ^* \phi(m)$ whenever $m \notin \mu(\succeq)$.

In other words, incentive compatible experiments induce the subject to announce truthfully, treating each decision problem as though it were in isolation. When there is no confusion, we drop the reference to $\mathscr{E}$ and simply refer to ϕ as incentive compatible.

Examples of Experiments

Our framework is quite general, covering nearly any choice-based, incentivized experiment.[8] Laboratory experiments, field experiments, consumer focus groups, and essentially any other incentivized choice elicitation procedure can all be represented in this way. Here we discuss several hypothetical examples to demonstrate how the theoretical framework maps into practical applications.

[8]Experiments that gather non-choice data such as neural activations, eye movements, pupil dilation, or galvanic skin response are excluded. If the decisions are strategic then one must also assume strategy choice can be represented as a single-person decision problem.

A Laboratory Choice Experiment. A researcher is interested in correlating giving in a dictator game with subjects' risk preferences. Each subject is endowed with \$100 and must choose how much to give (in dollar increments) to an anonymous recipient. Then the subject is presented a table with ten risky lotteries in the first column and ten safe lotteries in the second. They are asked to pick one lottery from each row—as in Holt and Laury (2002)—to reveal their risk attitudes. Subjects are paid their choice in the dictator game (with some money potentially going to an anonymous beneficiary) as well as for the realization of *one* randomly-chosen choice from the Holt-Laury risk elicitation procedure. Formally, $D_1 = \{(\$100,\$0),(\$99,\$1),\ldots,(\$0,\$100)\}$, and each remaining D_i ($i \in \{2,\ldots,11\}$) consists of the risky and safe lottery from row $i-1$ of the Holt-Laury table. The payment mechanism ϕ can be thought of as partitioning Ω into ten events, Ω_1 through Ω_{10}. For example, if one of ten bingo balls are drawn from a cage, then Ω_3 is the event that the third ball is drawn. For any given m and realization $\omega \in \Omega_j$ (meaning the jth ball was drawn), the mechanism ϕ pays both m_1 and m_{j+1}. With this payment mechanism, it is possible that the wealth effect from the dictator game earnings will reduce the subject's risk aversion, shifting her choices in the risk elicitation decisions. Thus, this experiment is not incentive compatible. Similarly, paying for multiple chosen lotteries is not incentive compatible due to portfolio effects. We show below that the only incentive compatible payment mechanism randomly chooses either the dictator game or one of the lottery choices for payment.

A Game Theory Experiment. Subjects play a symmetric, two-by-two, normal-form game against another, anonymous subject. Their strategies are U and D. They are also asked to guess which strategy they believe their opponent will play: L or R. The game pays \$1 to both subjects if the outcome is (U,L) or (D,R), and \$0 otherwise. Subjects also receive \$1 if they guess their opponent's action correctly, and \$0 otherwise. Here, $D_1 = \{U,D\}$ are the possible choices of one's own strategy, and $D_2 = \{L,R\}$ are the possible guesses of the other's strategy. If a subject views the opponent as having a 75% chance of playing L, then his truthful message is $m_1 = U$ and $m_2 = L$. Since both decisions are paid, the truthful message results in a lottery that pays \$2 with 75% probability and \$0 with 25% probability. But a deceitful message of $m' = (U,R)$ pays \$1 with certainty. A sufficiently risk-averse subject will prefer the guaranteed payoff, so this experiment is not incentive compatible.

A Focus Group. A large snack food manufacturer (Company A) invites a focus group to compare the company's products against those of its competitor (Company B). After sampling various products, consumers are asked if they prefer to take home a bag of Company A's potato chips or a bag of Company B's potato chips. Then consumers are asked if they prefer to take home Company A's chip dip or that of Company B. Thus, $D_1 = \{\text{Chip}_A, \text{Chip}_B\}$ and $D_2 = \{\text{Dip}_A, \text{Dip}_B\}$. Consumers do not get paid for both choices. Instead, a coin is flipped. If the coin lands 'heads', the consumers take home their choice of chips ($\omega \in H$ implies $\phi(m)(\omega) = m_1$). If the coin lands 'tails', they take home their choice of dips ($\omega \in T$ implies $\phi(m)(\omega) = m_2$). Since there is no state of the world where both goods are consumed, complementarities between goods will not affect the consumers' choices. This mechanism—called the Random Decision Selection (RDS) mechanism—is therefore incentive compatible, assuming a monotonicity axiom described below.

Incentive Compatibility and Monotonicity

We first consider the case where the researcher is unwilling to make *any* assumptions about subjects' admissible extensions.

Proposition 1. If every extension is admissible, then there exists an incentive compatible payment mechanism if and only if $k = 1$ (the experiment has only one decision problem).

Proofs not provided in the text appear in appendix.

Almost every interesting experiment requires at least two data points, and therefore at least two decision problems. Proposition 1 verifies that incentive compatibility is never free: Such experiments cannot be done without making some assumptions about subjects' extensions. This implies that every multiple-decision experiment necessarily represents a joint test of the both the main research question and the assumptions necessary to guarantee incentive compatibility of the payment mechanism.

One natural restriction on extensions is that they respect dominance. Given the underlying preference $\succeq$ on X, act f is said to *dominate* act g (written $f \sqsupseteq g$) if, for every $\omega \in \Omega$, $f(\omega) \succeq g(\omega)$. If $f \sqsupseteq g$ and $f(\omega) \succ g(\omega)$ for some ω then f strictly dominates g ($f \sqsupset g$).[9] An extension that respects the dominance relation is said to be *monotonic*.

Axiom 1 (Monotonicity). The extension $\succeq^*$ is a monotonic extension of $\succeq$ if $f \sqsupseteq g$ implies $f \succeq^* g$, and $f \sqsupset g$ implies $f \succ^* g$.

[9]Technically, $\sqsupseteq$ and $\sqsupset$ depend on $\succeq$. Because it will always be obvious, we use a notation which suppresses this dependence.

For each $\succeq$, let $\mathscr{E}^{\text{mon}}(\succeq)$ be the set of all monotonic extensions of $\succeq$.[10] All of our results concern the case where only monotonic extensions are admissible ($\mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$). Here, a sufficient condition for incentive compatibility is that acts resulting from truth-telling messages dominate all acts resulting from any other message, with strict dominance whenever the other message is not truthful. This fact is used throughout our paper, so we prove it formally in the following lemma.

Definition 2 (Truth Dominates Lies). A mechanism ϕ has the *truth dominates lies property* if, for every $\succeq$, every $m^* \in \mu(\succeq)$, and every $m \in M$, we have that $\phi(m^*) \sqsupseteq \phi(m)$, with $\phi(m^*) \sqsupset \phi(m)$ whenever $m \notin \mu(\succeq)$.

Lemma 1. If $\mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$ for every $\succeq$ and ϕ has the truth dominates lies property then ϕ is incentive compatible with respect to $\mathscr{E}$.

For some of the results we need in addition to assume that *every* monotonic extension is admissible ($\mathscr{E}(\succeq) = \mathscr{E}^{\text{mon}}(\succeq)$), or at least that the class of admissible extensions is "sufficiently rich". In this case, the truth dominates lies property of a mechanism is not only sufficient but also necessary for incentive compatibility.

Definition 3. A set of admissible extensions $\mathscr{E}$ is *rich* (or, satisfies *richness*) if $\mathscr{E} \subseteq \mathscr{E}^{\text{mon}}$ and, for any two acts $f, g \in X^\Omega$, $f \succeq^* g$ for every $\succeq^* \in \mathscr{E}(\succeq)$ implies $f \sqsupseteq g$.

Remark. The richness requirement requires that each $\mathscr{E}(\succeq)$ be sufficiently large so that if f is preferred to g for every admissible extension, then we can conclude that f dominates g. This is satisfied in many standard domains of extensions, including the following cases:[11]

- Every monotonic extension is admissible ($\mathscr{E} = \mathscr{E}^{\text{mon}}$).
- Every (subjective) expected utility extension is admissible.
- For every state ω, there is an admissible expected utility extension that puts subjective probability on ω arbitrarily close to one.
- Every probabilistically sophisticated extension states is admissible.
- Every multiple priors extension is admissible.

Under richness, the following useful lemma is immediate from the definitions.

Lemma 2. Suppose $\mathscr{E}$ satisfies richness. If a mechanism ϕ is incentive compatible with respect to $\mathscr{E}$, then it has the truth dominates lies property.

[10]This definition of monotonicity implicitly assumes that all states $\omega \in \Omega$ are non-null under each $\succeq^* \in \mathscr{E}^{\text{mon}}(\succeq)$, which essentially means that the subject views every state as having some (possibly tiny) chance of occurring. See Savage (1954) for a formal definition.

[11]Assuming no null states.

III: The Random Decision Selection Mechanism

Simply put, a random decision selection (RDS) mechanism randomly selects one of the k decision problems and pays the subject their announced choice from that problem. To our knowledge, this was first suggested in writing by Allais (1953) as a way of avoiding complementarities when eliciting multiple decisions. The following definition formalizes this mechanism in our framework.

Definition 4 (Random Decision Selection Mechanisms). A payment mechanism ϕ is a *random decision selection* (RDS) mechanism if there is a fixed partition $\{\Omega_1,\dots,\Omega_k\}$ of Ω with each Ω_i non-empty such that, for each $r \in \{1,\dots,k\}$ and $m \in M$, $\omega \in \Omega_r$ implies that

$$\phi(m)(\omega) = m_r.$$

In this section, we identify the exact restrictions on extensions needed to guarantee the incentive compatibility of this mechanism. We show that incentive compatibility of the RDS mechanisms obtains if and only if a weakened version of monotonicity—called ϕ-monotonicity—is assumed. ϕ-monotonicity requires that $\succeq^*$ respect dominance only when comparing a payment that results from a truthful report to the payments that could result from other reports.

Axiom 2 (ϕ-Monotonicity). Fix an experiment (D,ϕ). For any preference $\succeq$, the extension $\succeq^*$ is ϕ-monotonic if, for every $f \in \phi(\mu(\succeq))$ and $g \in \phi(M)$, $f \sqsupseteq g$ implies $f \succeq^* g$, and $f \sqsupset g$ implies $f \succ^* g$.[12]

Let $\mathscr{E}^{\phi\text{mon}}$ identify the ϕ-monotonic extensions for each $\succeq$. We now demonstrate that ϕ-monotonicity is equivalent to incentive compatibility of the RDS mechanisms.

Theorem 1. An RDS mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if and only if $\mathscr{E} \subseteq \mathscr{E}^{\phi\text{mon}}$.

Since $\mathscr{E}^{\text{mon}} \subseteq \mathscr{E}^{\phi\text{mon}}$, we have the following useful corollary.

Corollary 1. If $\mathscr{E} \subseteq \mathscr{E}^{\text{mon}}$ then all RDS payment mechanisms are incentive compatible with respect to $\mathscr{E}$.

The ϕ-monotonicity assumption is a form of local monotonicity that applies only for specific acts which depend on the particular experiment under consideration. If one requires ϕ-monotonicity for all possible experiments that use the RDS mechanism then the (global) monotonicity axiom is implied.

[12]For any set $S \subseteq M$, $\phi(S) = \{\phi(m)\}_{m\in S}$.

Proposition 2. Fix X, $\succeq$, and extension $\succeq^*$. If $\succeq^*$ satisfies ϕ-monotonicity for every experiment (D,ϕ) in which ϕ is an RDS mechanism, then $\succeq^*$ satisfies monotonicity.

Alternatively, the intersection over all $\mathscr{E}^{\phi\text{mon}}$ such that ϕ is an RDS mechanism is exactly $\mathscr{E}^{\text{mon}}$. Therefore, ϕ-monotonicity is sufficient to guarantee incentive compatibility in any *one* experiment using the RDS mechanism, but the (global) monotonicity assumption is necessary to guarantee incentive compatibility in *all* such experiments.

IV: A Characterization of Incentive Compatible Mechanisms

In this section, we assume that preferences over X are always strict. This is a weak assumption, since preferences over a finite set X are strict 'generically'. With strict preferences, the most-preferred element in each decision problem is unique ($|\mu_i(\succeq)| = 1$). In that case, Lemmas 1 and 2 would be modified slightly to say that (D,ϕ) is incentive compatible with respect to a rich $\mathscr{E}$ if and only if, for every preference $\succeq$ and every message $m \neq \mu(\succeq)$,

$$\phi(\mu(\succeq)) \sqsupset \phi(m).$$

Another consequence of assuming strict preferences is that there may be messages that cannot be truthful for any preference relation. For example, if $D_1 = \{x,y\}$, $D_2 = \{y,z\}$, and $D_3 = \{z,x\}$, then $m = (x,y,z)$ cannot be *rationalized* by any $\succeq$, since it would imply $x \succ y \succ z \succ x$. We therefore need to distinguish messages that can be rationalized from those that cannot. Let

$$M_R = \left\{m \in M : (\exists \succeq)\ m \in \mu(\succeq)\right\}$$

be the set of *rationalizable* messages. $M_{NR} = M \setminus M_R$ is then defined as the set of non-rationalizable messages. One immediate necessary condition for incentive compatibility with respect to a rich $\mathscr{E}$ is that all non-rationalizable messages be dominated by the truthful message.

To understand how incentive compatibility can extend beyond the RDS mechanism, consider a mechanism ϕ and suppose for the moment that the subject's submitted announcement vector m^* is truthful, meaning $m_i^* = \mu_i(\succeq)$ for each i. Given any other set $E \subseteq X$, we can also ask whether m^* reveals the subject's true favorite element in E. If ϕ pays this 'inferred favorite' element from E in some state of the world, incentive compatibility may still be maintained.

To construct such mechanisms, we must first characterize the sets E whose favorite element can always be inferred from the subject's choices. To do this, we must identify the preferences that are revealed by any truthful announcement m.

Definition 5. Fix any rationalizable message $m \in M_R$. For two distinct choice objects $x, y \in X$, say that x is *directly revealed preferred* to y under announced choices $m = (m_1, \dots, m_k)$ if there is $1 \le i \le k$ such that $m_i = x$ and $y \in D_i$. Denote the transitive closure of this relation by $R(m)$, and say that x is *revealed preferred* to y under choices m if $xR(m)y$.

The relation $R(m)$ is transitive and asymmetric since $m \in M_R$, but it need not be complete. Denote by $L(x,m) = \{y \in X : xR(m)y\}$ and $U(x,m) = \{y \in X : yR(m)x\}$ the sets of elements that are revealed to be worse than x and better than x under choices m, respectively. Clearly, $L(x,m) \subseteq L(x,\succeq)$ and $U(x,m) \subseteq U(x,\succeq)$ when $m = \mu(\succeq)$, with strict inclusions for some x when $R(m)$ is not a complete relation.

The following lemma is just a statement of the well-known fact that a choice function is rationalizable if and only if it satisfies a version of the strong axiom of revealed preference. For a proof, see Richter (1966).

Lemma 3. Announced choices are rationalizable ($m \in M_R$) if and only if $R(m)$ is acyclic.

Let

$$\text{dom}_m(E) = \{x \in E : (\forall y \in E \setminus \{x\}) \;\; xR(m)y\}$$

be the set of $R(m)$-*dominant* elements of E. If E has no element that is $R(m)$-dominant (meaning m does not reveal the most-preferred element of E), then $\text{dom}_m(E) = \emptyset$. Otherwise, $\text{dom}_m(E)$ contains a unique element since preferences are strict. If $m = \mu(\succeq)$, then either $\text{dom}_m(E) = \emptyset$ or else $\text{dom}_m(E) = \text{dom}_{\succeq}(E)$.

We can now describe the sets $E \subseteq X$ whose most-preferred elements are always revealed when the subject submits a truthful message.

Definition 6 (Surely Identified Sets). A non-empty set $E \subseteq X$ is *surely identified (SI)* if, for every $m \in M_R$,

$$\text{dom}_m(E) \neq \emptyset.$$

In other words, E is SI if, for any order $\succeq$, the message $m = \mu(\succeq)$ identifies the most-preferred element of E, so that $\text{dom}_m(E) = \text{dom}_{\succeq}(E)$.

Let $SI(\mathscr{D})$ be the collection of sets surely identified from the given set of decision problems $\mathscr{D}$.[13] Obviously, any D_i is in $SI(\mathscr{D})$, but there may be other sets in $SI(\mathscr{D})$. For instance, if $D_1 = \{x,y\}$, $D_2 = \{y,z\}$, and $D_3 = \{z,x\}$, then $\{x,y,z\} \in SI(\mathscr{D})$. Also, any singleton set $\{x\}$ is trivially SI. As another illustrative example, if every subset of X of

[13]Recall that $\mathscr{D} = \{D_1, \dots, D_k\}$ is the collection of decision problems, while $D = (D_1, \dots, D_k)$ is the ordered list of decision problems.

cardinality n is in $\mathscr{D}$, then every subset of cardinality more than n is in $SI(\mathscr{D})$. A full characterization of surely identified is given by the following proposition.

Proposition 3. $E \in SI(\mathscr{D})$ if and only if E is either a singleton, or for every pair $\{x, y\} \subseteq E$, there exists $D \in \mathscr{D}$ for which $\{x, y\} \subseteq D \subseteq E$.

We wish to discuss mechanisms that choose sets in $SI(\mathscr{D})$ for payment; to this end, define the *payment set* of a mechanism ϕ at each state ω by

$$P^{\phi}(\omega) = \{\phi(m)(\omega)\}_{m \in M},$$

and the collection of all payment sets by

$$\mathscr{P}^{\phi} = \{P^{\phi}(\omega)\}_{\omega \in \Omega}.$$

In an RDS mechanism, $\mathscr{P}^{\phi} = \mathscr{D}$. The following definition generalizes RDS mechanisms to allow other surely identified sets to be used as payment sets.

Definition 7 (Random Set Selection Mechanisms). A mechanism ϕ is a *random set selection* (RSS) mechanism if

(1) $\mathscr{P}^{\phi} \subseteq SI(\mathscr{D})$, and
(2) if $m \in M_R$ then for each $\omega \in \Omega$, $\phi(m)(\omega) = \text{dom}_m(P^{\phi}(\omega))$

The first condition requires that every payment set be surely identified, and the second requires that most-preferred elements are chosen from each payment set whenever messages are rationalizable. No restrictions are placed on the acts chosen at non-rationalizable messages.

In an RDS mechanism, each payment set is a decision problem, and each decision problem corresponds to at least one payment set ($\mathscr{D} = \mathscr{P}^{\phi}$). Since decision problems are surely identified, and since each m_i is the revealed-most-preferred outcome in D_i, RDS mechanisms are special cases of RSS mechanisms.[14] Our main theorem shows that a particular subclass of RSS mechanisms (which includes the RDS mechanisms) fully characterizes the set of incentive compatible mechanisms when the set of admissible extensions is rich.

Theorem 2. Let the admissible extensions $\mathscr{E}$ satisfy richness. A mechanism ϕ is incentive compatible with respect to $\mathscr{E}$ if and only if it is a random set selection (RSS) mechanism in which

[14]RDS mechanisms completely specify the payments from non-rationalizable messages, while RSS mechanisms do not. Thus, the family of RSS mechanisms for which $\mathscr{P}^{\phi} = \mathscr{D}$ can be strictly larger (in the sense of inclusion) than the family of RDS mechanisms.

(1) $D_i \in SI(\mathscr{P}^\phi)$ for each $D_i \in \mathscr{D}$, and
(2) $\phi(M_R) \cap \phi(M_{NR}) = \emptyset$.

In other words, a mechanism is incentive compatible if and only if it is an RSS mechanism (rationalizable messages pay the most-preferred element from some randomly-selected, surely-identified set); each decision problem can be surely identified from the payment sets; and non-rationalizable messages map to different acts than rationalizable messages.

The intuition for the above result is quite simple. First, to show that any such mechanism is incentive compatible, recall that incentive compatibility is satisfied when an agent with a preference $\succeq$ receives the best option in each state of the world according to $\succeq$. But this is precisely how our mechanisms are defined: ϕ chooses, for each m, an optimal element for any $\succeq$ which rationalizes m. Furthermore, condition (1) of the theorem guarantees that, if the subject misrepresents his preferences in any one of the decision problems, then this will affect the resulting output of the mechanism. Since truth-telling gives the best outcome in every state, this change will result in a dominated act. Condition (2) guarantees that reporting non-rationalizable messages is never optimal. The proof of the other direction is again quite simple, since incentive compatibility almost directly implies that the criteria for an RSS mechanism as well as conditions (1) and (2) of the theorem are satisfied.

When RDS Mechanisms Are the Only Incentive Compatible Mechanisms

In an RDS mechanism, the collection of payment sets coincides with the collection of decision problems ($\mathscr{P}^\phi = \mathscr{D}$). Theorem 2 shows that non-RDS mechanisms (in which $\mathscr{P}^\phi \neq \mathscr{D}$) can be incentive compatible. Such mechanisms are almost never observed in practice, suggesting that most experiments naturally rule out such mechanisms. We now identify conditions under which these other incentive compatible mechanisms are in fact ruled out.

First, if the decision problems are sufficiently distinct and small in number, then announced choices will not reveal the most-preferred element of any other set $E \notin \mathscr{D}$. In other words, every surely identified set will either be a decision problem or a singleton. Since every incentive compatible mechanism is an RSS mechanism (Theorem 2), and the payment sets of an RSS mechanism are all surely identified, it follows that $\mathscr{P}^\phi \subseteq \mathscr{D}$.[15]

[15]There may be singleton payment sets which are not included in $\mathscr{D}$. We ignore this insignificant complication in this informal discussion.

Furthermore, it seems unlikely that an experimenter would want to give subjects a *redundant* decision problem—one that is surely identified by the remaining decision problems—, since there is no new information that can be obtained from that choice. Using condition (1) of Theorem 2, this implies that every decision problem must be a payment set at some state, that is $\mathscr{P}^{\phi} \supseteq \mathscr{D}$. Combining these two observations, we get that $\mathscr{P}^{\phi} = \mathscr{D}$, which essentially means that any incentive compatible mechanism is an RDS mechanism.

To formalize this intuition we need the following definition.

Definition 8 (Redundant Decision Problems). Fix $D = (D_1, \ldots, D_k)$. The decision problem D_i is redundant in D if $D_i \in SI(\mathscr{D} \setminus \{D_i\})$. Say that D_i is non-redundant otherwise. The collection of all non-redundant decision problems is denoted $\text{NRed}(\mathscr{D})$

We say that a payment mechanism ϕ is *essentially identical to an RDS mechanism* if there is an RDS mechanism ϕ^* such that $\phi(m)(\omega) = \phi^*(m)(\omega)$ for every $m \in M_R$ and ω at which $\mathscr{P}^{\phi}(\omega)$ is non-singleton. Such a mechanism can differ from an RDS mechanism in at most two ways: (1) ϕ may pay differently for non-rationalizable messages, and (2) there may be some states ω where ϕ pays a constant payment regardless of m.

Corollary 2. Let $\mathscr{E}$ satisfy richness. The following conditions are equivalent:

(1) $SI(\mathscr{D}) \setminus \{\{x\}_{x \in X}\} = \text{NRed}(\mathscr{D})$
(2) If ϕ is incentive compatible with respect to $\mathscr{E}$, then ϕ is essentially identical to an RDS mechanism.

In words, if the decision problems are non-redundant and there are no other non-singleton surely-identified sets, then the only incentive compatible mechanisms are essentially identical to RDS mechanisms. The following corollary gives a more easily-tested sufficient condition for RDS mechanisms to be the only incentive compatible mechanisms.

Corollary 3. Suppose $\mathscr{E}$ is rich and D is such that $M_{NR} = \varnothing$. If ϕ is incentive compatible with respect to $\mathscr{E}$ then it is essentially identical to an RDS mechanism.

Environments in which Corollary 3 are satisfied abound in experiments. For example, any experiment in which the D_i are pairwise disjoint satisfy the hypothesis. Since most experimental designs satisfy the conditions of this corollary, RDS mechanisms are, in practice, the only incentive compatible mechanisms.

V: Paying for Multiple Decisions

Theorem 2 has an important immediate corollary: If a mechanism is incentive compatible for a rich set $\mathscr{E}$, and if there is a state $\omega \in \Omega$ and a message m for which $\phi(m)(\omega) \notin \bigcup_{i=1}^{k} D_i$, then for any other message m', it must be the case that $\phi(m')(\omega) = \phi(m)(\omega)$. In other words, the payment set at ω must be a singleton.

Why is this observation important? Experimental economists often pay subjects for *every* decision made in an experiment. That is, if the message $m = (m_1, \ldots, m_k)$ is announced, experimentalists often pay subjects the *bundle* $\{m_1, \ldots, m_k\}$. This bundle is typically not itself an element of any D_i. This tells us right away that such mechanisms are not incentive compatible without assuming further restrictions on the domain of preferences. Essentially, complementarity effects between the objects of choice can lead to distortions in the announced messages.

Before exploring restrictions on preferences, we note that there is one way in which a bundle can be paid without distorting incentives: If it is paid regardless of the subject's message. This is formalized in the following definition.

Definition 9. A mechanism ϕ is *invariant* on a set $E \subseteq X$ if, for every $\omega \in \Omega$ and $m, m' \in M$, $\phi(m)(\omega) \in E$ implies that $\phi(m')(\omega) = \phi(m)(\omega)$.

The following result formalizes the claim that, if one wants incentive compatibility, the payment of a bundle (or, *anything* outside of $\cup_i D_i$) can only be done if it is not affected by the subject's message.

Corollary 4. If $\mathscr{E}$ is rich and a mechanism ϕ is incentive compatible with respect to $\mathscr{E}$, then ϕ is invariant on $X \setminus \bigcup_{i=1}^{k} D_i$.

Justifying Bundle Payments

Despite the above results, paying for multiple decisions is common practice in economics experiments (see Table I). Corollary 4 shows that this cannot be justified on incentive grounds when every preference over bundles is admissible. Something must be assumed if this practice is to be justified. We now explore how strong such assumptions must be in order to guarantee incentive compatibility.

Up until now, we have assumed X is an abstract set of alternatives. Here, we wish to allow agents to consume packages of goods; for example, if they choose beer from {beer, tequila}, and chicken from {chicken, beef}, then they consume a joint package consisting of beer and chicken. Analogously, if they choose pie from {pie, cake}, and pie from {pie, ice cream}, then they consume a package consisting of *two* slices of pie–one for each decision problem in which pie is chosen.

To this end, we model consumption space as $X = \mathbb{Z}_+^Y$–a space of commodity bundles of elements of Y with integer units (goods are assumed to be discrete).[16] Thus, any bundle of elements is described by a vector $x = (x_y)_{y \in Y} \in X$, where x_y is the number of units from the good $y \in Y$ in the bundle. For example, if $Y = \{a, b, c\}$, then $x = (1,0,0)$ refers to the bundle where the subject receives only one unit of good a. With this notation it is meaningful to talk about the addition of bundles: If $x, x' \in X$ then $x + x'$ is the bundle defined by $(x+x')_y = x_y + x'_y$.

With this framework, we can assume that each D_i consists only of unit vectors x for which $\sum_y x_y = 1$. Thus, the subject is asked to choose among menus of single items in Y. Each announced choice $m_i \in D_i$ is therefore also a unit vector. If the payment $\phi(m)(\omega)$ is not a unit vector, then a bundle outside if $\cup_i D_i$ is being paid, and incentive compatibility may be violated.

Almost all experiments that pay in bundles can be characterized as randomly choosing a subset of the decision problems (or, all decision problems) and paying the bundle of messages announced in those chosen problems. We refer to these as Random Multiple Decision Selection (RMDS) mechanisms. To formalize these mechanisms, let $\mathscr{I}$ be the set of all subsets of $\{1, \ldots, k\}$. An RMDS mechanism randomly selects one set of decision problems $F \in \mathscr{I}$ for payment.

Definition 10 (Random Multiple Decision Selection Mechanisms). For any experiment (D, ϕ), the payment mechanism ϕ is a *random multiple decision selection* (RMDS) mechanism if there is a function $I : \Omega \to \mathscr{I}$ mapping each state $\omega \in \Omega$ into a subset $I(\omega) \in \mathscr{I}$ of decision problems such that, for each $m \in M$,

$$\phi(m)(\omega) = \sum_{i \in I(\omega)} m_i.$$

An RDS mechanism is a special case of an RMDS mechanism where only single decision problems are chosen, so that $|I(\omega)| = 1$ for each ω. When all decision problems are paid, $I(\omega) = \{1, \ldots, k\}$ for every ω.

We are interested in understanding what exactly needs to be assumed on preferences in order to make a given RMDS mechanism incentive compatible with respect to some rich $\mathscr{E}$. The following definition is useful for providing the answer.

[16] The notation $\mathbb{Z}$ refers to the integers. The result below would also hold in the environment where $X = \mathbb{R}_+^Y$. Discreteness is maintained to mesh with the previous analysis.

Definition 11 (No Complementarities at the Top). Let $\mathscr{F} \subseteq \mathscr{I}$ be a collection of non-empty subsets of $\{1,\ldots,k\}$. The preference $\succeq$ over X satisfies no complementarities at the top (NCaT) relative to $\mathscr{F}$ if, for every $F \in \mathscr{F}$ and every $m \in M$,

$$\sum_{i \in F} \mu_i(\succeq) \succeq \sum_{i \in F} m_i,$$

with a strict preference if there exists $i \in F$ for which $\mu_i(\succeq) \succ m_i$.

The following proposition provides a necessary and sufficient condition on preferences for a given RMDS mechanism to be incentive compatible.

Proposition 4. Given is an experiment (D,ϕ), where ϕ is an RMDS mechanism with associated function $I : \Omega \to \mathscr{I}$. Then ϕ is incentive compatible with respect to a rich $\mathscr{E}$ if and only if any admissible preference $\succeq$ on X satisfies NCaT with respect to $I(\Omega)$.

Proof. Apply the definition of incentive compatibility. □

Proposition 4 has a straightforward interpretation. Pick any state ω and consider the set of chosen decision problems $I(\omega) \subseteq \{1,\ldots,k\}$. Imagine the subject consuming the bundle of goods she considered optimal for each D_i, where $i \in I(\omega)$. Then this bundle must be preferred to any other possible bundle of goods from those decision problems. For example, consider the problems $D_1 = \{\text{Reese's cup}, \text{hot dog}\}$ and $D_2 = \{\text{milk}, \text{beer}\}$, and suppose $I(\omega) = \{1,2\}$, so both decisions get paid at state ω. If a Reese's cup is preferred to the hot dog, and beer is preferred to milk, then consuming the Reese's cup with beer must be preferred to consuming the Reese's cup with milk, or the hot dog with beer. In this sense, NCaT with respect to $I(\Omega)$ formalizes a lack of complementarities between the items in the decision problems that can be chosen for payment.

Show-Up Fees

Experimenters often pay subjects a (state-independent) show-up fee for their participation, in addition to their earnings from their choices. This is frequently necessary to solicit sufficient participation. Technically, paying a show-up fee turns each payment into a bundle. For example, if x_0 is the show-up fee and an RDS mechanism is used, then the realized payment is the bundle $x_0 + m_i$ when decision problem D_i is randomly chosen. Without ruling out any form of complementarities, it is possible that the show-up fee may distort incentives. As an extreme example, paying a \$1,000 show-up fee is likely to generate wealth effects and reduce risk aversion.

In practice, however, we agree that (relatively small) show-up fees are unlikely to distort incentives. Formally, this is equivalent to assuming that $x \succeq y$ implies $(x_0 + x) \succeq (x_0 + y)$ for every $x, y \in X$. This assumption represents a very weak form of translation invariance—restricted only to additions by x_0—and could be termed *show-up fee invariance*.

VI: Choices Over Uncertain Prospects

Thus far, the choice objects in X have been modeled abstractly, with no particular structure. Random choices of $x \in X$ are then modeled as acts. Here we discuss the special case where each $x \in X$ is itself a random prospect, such as an act or lottery. In this setting, there are two plausible conditions that lead to violations of monotonicity and, therefore, incentive compatibility of an RDS mechanism: the randomizing devices may appear to have correlation, and subjects with non-expected-utility preferences may reduce compound lotteries to simple lotteries.

Correlated Randomizing Devices

Let $X = Z^S$ be a space of acts, where S is a state space and Z is an outcome space. If Ω is the state space corresponding to mechanism ϕ, then the objects of payment are elements of the space $(Z^S)^\Omega$, or $Z^{S\times\Omega}$. A subject may believe that there is 'correlation' between the processes generating S and Ω, whether or not this correlation is justified empirically. For example, suppose D_i is chosen for payment if it is sunny tomorrow, and $x, y \in D_i$ are acts whose payments depend on the temperature tomorrow. Clearly, subjects picking between x and y should condition on the fact that they will only be paid for this choice in the event that it is sunny. This conditioning may alter preferences over x and y. This would represent a violation of monotonicity in our general framework. In general, if preferences over X can depend on the realization of $\omega \in \Omega$, then incentive compatibility can fail.

Since the experimenter can choose the randomizing device used to choose D_i, care should be taken to ensure that the subject will behave as if S and Ω are independent. If S refers to the weather tomorrow in Detroit, then it might make sense to let Ω refer to the outcome of a coin flip or a computer-drawn random number.

An interesting discussion of this sort of independence between S and Ω is undertaken by Bade (2012). In our work, we think of S (the ambient probability space) as a primitive, and the experimenter constructs Ω as part of the experimental design. Ultimately, subjects' payoffs are then acts with state space $S \times \Omega$. Bade assumes the ambient probability space and the space corresponding to the randomization device are one and the same (mathematically, this generalizes our approach because, in particular, she is allowed to assume that the state space is a product space). We can refer to the state space in her environment simply as S^*. An RDS mechanism in this framework then works as follows: Each decision problem D_i is associated with an event $S_i \in S^*$. The state of nature s^* is realized, and the decision maker is paid $f(s^*)$, where $s^* \in S_i$ and $f \in D_i$ is the act chosen from D_i.

Bade's interest is in measurability conditions which guarantee incentive compatibility. She fixes a preferences and then relates the σ-algebra generated by decision problems to the σ-algebra generated by the mechanism. A behavioral condition of independence (for the fixed preference) with respect to these two σ-algebras is identified for the preference under consideration. For this preference, this condition is equivalent to a weak form of incentive compatibility, stating that if a collection of acts is chosen by the subject, then each of those acts must be optimal for the individual decision problems.

Instead of fixing preferences and varying σ-algebras, we fix the σ-algebras and study restrictions on preferences. Formally, the σ-algebras implicit in our work are the two generated by the sets $\{\{s\} \times \Omega\}$ and $\{S \times \{\omega\}\}$ respectively. We then identify the necessary and sufficient conditions on *preferences* for the RDS mechanism to be strongly incentive compatible. The two approaches are complementary and help to understand the general structure of incentive compatibility in abstract environments.

Reduction of Compound Lotteries

In this paper we assume subjects view uncertain prospects as acts, so that subjects need not have beliefs representable by probabilities. This general framework allows the inclusion of essentially any kind of preferences over prospects, including non-expected utility models and preferences exhibiting ambiguity aversion.

Holt (1986) and Karni and Safra (1987), in commenting on the preference reversal experiments of **?** (among others), suppose that subjects view both the choice objects and the RDS mechanism as lotteries with well-defined probabilities. Here, we can think of an RDS mechanism as generating two-stage lotteries, with the choice of which decision to pay being the 'upper stage' (evaluated by $\succeq^*$), and the actual lotteries chosen from each D_i being the 'lower stage' (evaluated by $\succeq$). These authors assume subjects evaluate these compound lotteries as being equal to an equivalent one-stage lottery. This represents a particular restriction on $\mathscr{E}(\succeq)$. In this setting, they show by examples that if subjects' upper-stage preferences violate the expected utility axioms (for example, if they have rank-dependent preferences), then the RDS mechanism is not incentive compatible. Similar arguments are found in Cox et al. (2011) and Harrison and Swarthout (2011).

These examples appear to contradict our result that (upper-stage) monotonicity is all that is needed for an RDS mechanism to be incentive compatible. But the apparent contradiction is resolved by the following observation:

Observation 1. In the lotteries framework, if $\succeq^*$ satisfies (upper-stage) monotonicity and the reduction of compound lotteries, then $\succeq$ satisfies the (lower-stage) independence axiom.

Conversely, if reduction of compound lotteries is assumed, then violations of the (lower-stage) independence axiom imply violations of (upper-stage) monotonicity. In the examples referenced above, the resulting monotonicity violations are also violations of ϕ-monotonicity (for a given RDS mechanism ϕ), so incentive compatibility fails.

This observation shows the strength of the reduction axiom. For example, a subject with rank-dependent utilities who satisfies reduction will prefer an act that is strictly dominated (in the upper stage) by another act.

In a companion paper (Azrieli et al., 2012), we analyze incentives in experiments when subjects view uncertain prospects as objective lotteries. In addition to providing a larger class of incentive compatible mechanisms, we show that if all rank-dependent preferences are admissible and subjects reduce compound lotteries, then there are experiments for which no incentive compatible mechanism exists. On a positive note, Cox et al. (2011) find a mechanism that is incentive compatible under linear cumulative prospect theory (Schmidt and Zank, 2009) when reduction is assumed and lotteries are cosigned.[17] Thus, incentive compatible mechanisms can exist in some settings where (upper-stage) monotonicity fails. Given proposition 1 above, however, different mechanisms will have to be found for different domains of non-monotonic extensions.

VII: Applying the Results in Experimental Design

If an experimenter believes that subjects' preferences over gambles satisfy monotonicity (meaning they never choose dominated gambles), then our recommendation is simple:

[17] A set of lotteries is cosigned if, in each state, either all lotteries give positive payments, or all lotteries given negative payments.

(1) **Carefully define the decision problems.** An experiment likely consists of many choices made by each subject. A *decision problem* is defined as a set of choices that will be analyzed as if they were taken independently of all other choices outside the set, though not necessarily independent of each other. Roughly, each decision problem represents a unit of observation. For example, if subjects are asked to play five 10-period repeated games, each against five different opponents, then each 10-period repeated game would (likely) constitute a single decision problem.

(2) **Pay for one randomly-chosen decision problem.** At the end of the experiment, randomly draw one (and only one) of the decision problems, and pay the subjects their choices in that one problem. We call this the *RDS mechanism*. Under the weak assumption of monotonicity (that preferences respect dominance), the RDS mechanism will induce subjects to analyze each decision problem as if it were in isolation.

There are, however, several comments and caveats that should be taken into account when applying these results to experimental design.

(1) **Paying for multiple decisions.** Many experimenters currently pay for several or all decision problems (see Table I). With such a mechanism, complementarities between decision problems (such as wealth effects or portfolio effects) may distort subjects' choices. This practice can be justified, however, if one explicitly assumes no complementarities between paid decision problems. This is formalized by the NCaT assumption given in definition 11. In practice, the appropriateness of the NCaT assumption will depend on the decision problems given.

(2) **Framing.** Under the monotonicity assumption, the RDS mechanism is incentive compatible. The elicited preferences, however, may be altered by the mere presence of other decision problems. For example, choices over gym memberships may be altered if subjects are asked in another decision problem to choose one of three cheeseburgers.[18] In this case, the RDS mechanism is still incentive compatible—meaning it will still elicit choices truthfully—, but those true preferences may have been altered by the experimental design. This is a framing effect. Framing is not an issue of incentives or of the payment mechanism, so we have little to say about mitigating its effects. Experimenters should simply be mindful when choosing their decision problems that framing effects may occur and may reduce the generalizability of their results.

[18]Cox et al. (2011) refer to this as a violation of the *isolation hypothesis*.

(3) **Learning.** When decisions are offered sequentially, it is possible that subjects 'learn how to choose' with experience. Formally, their preferences change during the experiment. If the order of decisions were changed, results would differ. This represents another example of a framing effect. If the researcher is explicitly interested in learning effects, then the sequence of choices should be considered as one large decision problem. If the choices are viewed as different decision problems then the RDS mechanism will technically be incentive compatible under monotonicity—eliciting true choices at every point—, but the generalizability of the results will be reduced. Again, this problem is not specific to the payment mechanism; it is an issue with the list of decision problems.

(4) **Monotonicity violations.** Monotonicity is generally regarded as one of the weakest axioms in decision theory. But there are still plausible scenarios where the monotonicity axiom may fail. In these cases, the RDS mechanism may no longer be incentive compatible. The following are examples of possible monotonicity violations.

(a) *Decision Overload*. This is a common issue in experimental design. Subjects are asked to make so many decisions that they do not expend much cognitive effort on each choice. This is particularly plausible when there are effort-minimizing choices m^0 to which subjects can default when cognitively overloaded. In an RDS mechanism, suboptimal choices of m_i^0 over the truthful favorite m_i^* would represent a violation of monotonicity. Thus, the (positive) appeal of the monotonicity axiom reduces as the number of decision grows. We also believe, however, that cognitive overload will have a detrimental impact on *any* payment mechanism. Simply put, paying subjects \$20 for 100 decisions may lead to noisy or biased results, regardless of the specific payment rule.

(b) *Reduction of Compound Lotteries and Non-Expected Utility*. Suppose the objects of choice are gambles. If gambles are viewed as objective lotteries, and if two-stage lotteries are reduced to simple lotteries in subjects' assessments, then monotonicity implies expected utility preferences. If the researcher is explicitly interested in studying non-expected utility models, but does insist that reduction of compound lotteries is plausible, then the RDS mechanism may no longer be incentive compatible. In that case, there may not exist any incentive compatible mechanism with more than one decision problem

(Azrieli et al., 2012), or other mechanisms specific to the model at hand must be found (as in Cox et al., 2011).

(c) *Correlated randomization.* If the choice objects are gambles (either acts or lotteries), then their outcomes are determined by some random process. The period chosen for payment in an RDS mechanism is also chosen by a random process. Our definition of monotonicity (thus, incentive compatibility of the RDS mechanism) requires that subjects not perceive certain correlations between these two random processes. Specifically, subjects should not change their choices after conditioning on the event that decision problem i was chosen for payment. We recommend using a physical randomization device such as a die roll or bingo cage to choose the period for payment, and a separate device (or separate, independent draw from the same device) to determine the payoffs in the chosen gambles.

(d) *Ex-Ante Fairness*. Suppose a subject is presented with two decision problems: (1) give \$10 to person A or B, and (2) give \$10 to person A or C. A coin flip will determine which problem is chosen. It may be that $A \succeq B$ and $A \succeq C$, but, because of ex-ante fairness, the subject prefers a coin flip between A and B over giving it to A with certainty. Such a choice would violate monotonicity and cause the RDS mechanism not to be incentive compatible. The NCaT assumption is also not particularly appealing in this case (\$10 each for A and B may be preferred to \$20 for A only), so paying for both decisions also may not be incentive compatible. One possible solution may be to pay the first decision with very high probability and the second with very low probability, but even this will violate incentive compatibility for some preferences. In general, experiments with difficult moral or ethical choices may be plagued by this persistent incentive problem.

(5) **The show-up fee.** Paying a fixed show-up fee in addition to subjects' earnings can create wealth effects that skew subjects' decisions. We view this as a reasonable practice, however, provided that the show-up fee is not unreasonably large. This can be justified by assuming *show-up fee invariance*: subjects' preferences aren't altered when every choice object is bundled with the show-up fee.

In most experiments with reasonable-sized show-up fees, the assumptions of monotonicity and show-up fee invariance appear to be reasonable. If a separate randomizing device is used to choose the paid decision, then the RDS mechanism will be incentive compatible. Furthermore, if the experimenter is not willing to assume more than monotonicity, then the RDS mechanism is essentially the *only* incentive compatible mechanism that can be used.

These results were not obtained under some complex, assumption-laden theory of behavior. Rather, our assumptions are extremely minimal: We assume that subjects have preferences that guide their choice, and that those preference extend to preferences over acts. We study a context where the experimenter believes those preferences over acts respect dominance, but little else. If more is known about preferences (such as the NCaT condition from definition 11), then more mechanisms may become incentive compatible.

VIII: Discussion

Positive results can only be obtained by making structural assumptions (recall Proposition 1). In this paper we study incentive compatibility with respect to rich $\mathscr{E} \subseteq \mathscr{E}^{\text{mon}}$, because we believe monotonicity to be one of the most basic and intuitive axioms of decision theory; it is entirely plausible that subjects who violate monotonicity would reverse their choice over gambles if shown the dominating gamble. It may still be of interest, however, to study incentive compatibility with respect to different families of extensions. In particular, additional axioms may be compelling, leading to a study of incentive compatibility with respect to other conditions on $\mathscr{E}$. For example, Cox et al. (2011) have found an alternative incentive compatible mechanism in a lotteries framework without monotonicity, assuming a certain structure on the admissible choice objects X. In general, how exactly the set of incentive compatible mechanisms will vary in $\mathscr{E}$ is an interesting open question.

Experiments typically only elicit a small number of choices. These choices do not reveal the entire preference relation; they only reveal equivalence classes of preferences that would make the same set of choices. In our general environment, the entire relation can only be elicited by offering every pair of elements as a different decision problem. In more structured environments, however, we can elicit an entire preference relation through one observable choice. When that is possible, we could then infer *any* collection of choices from *any* set of decision problems $\mathscr{D}$. In a sense, the preference elicitation question generalizes the preceding results to an arbitrary collection of decision problems, requiring in addition that only one choice be observed. Doing so in a strictly incentive compatible way (and under weak assumptions on extensions) would be particularly valuable, since single-choice experiments avoid entirely the problem of complementarities.

There are two well-known preference domains where entire preference relations can be elicited with a single choice. Consider first the case of $X = \mathbb{R}^n$ with linear preferences (risk-neutral expected utility preferences, for example) representable by the function $u(x) = p \cdot x$, where $p \in \Delta^n$ (the n-dimensional simplex). Eliciting preferences is equivalent to eliciting the vector p. A function $\phi : \Delta^n \to \mathbb{R}^n$ is a *scoring rule*. It is *proper* if for all $p \in \Delta^n$ and all $q \neq p$, it is the case that $p \cdot \phi(p) > p \cdot \phi(q)$. Proper scoring rules are similar to incentive compatible mechanisms, in that they incentivize subjects to announce their p (and, thus, their preferences) truthfully. A further exploration of scoring rules may help inform our analysis mechanisms that are incentive compatible *regardless* of the particular list of decision problems at hand.

The other preference domain is that of Cobb-Douglas preferences over $\mathbb{R}^n$, which have a utility representation of the form $u(x) = \prod_i x_i^{\alpha_i}$, where the α_i's are nonnegative and sum to one. It is well-known that we can infer the entire Cobb-Douglas preference relation through a single choice over the linear budget set with equal prices.

It remains an open question how these example domains could be generalized. Clearly, stronger assumptions than we make in this paper would be needed, but may eventually be justifiable through behavioral observations.

Appendix A: Proofs

A.1: Proof of Proposition 1

Let $x \in X$ denote both the choice object x, and the constant act that pays x at every state $\omega \in \Omega$. For sufficiency, if $k = 1$ then the mechanism in which $\phi(m) = m$ for each $m \in M$ is clearly incentive compatible. The proof of necessity proceeds in several steps. In each, assume the hypothesis that ϕ is incentive compatible and $\mathscr{E}$ has no restrictions.

Step 1: $|\text{Range}(\phi)| > 1$.

If $x, y \in D_i$ (with $x \neq y$) then consider a preference $\succeq$ where $x \succ z$ for all $z \neq x$ and a preference $\succeq'$ where $y \succ' z$ for all $z \neq x$. Let $m = \mu(\succeq)$ and $m' = \mu(\succeq')$, and note that $m \neq m'$ since $m_i = x$ and $m'_i = y$. Incentive compatibility therefore requires $\phi(m) \succ^* \phi(m')$, which implies $\phi(m) \neq \phi(m')$. Thus, $|\text{Range}(\phi)| > 1$.

Step 2: $\text{Range}(\phi) \subseteq X$ (that is, $\phi(m)$ is a constant act for every $m \in M$).

Suppose not. Then there is some $m' \in M$ such that $\phi(m')$ is not a constant act. Using step 1, let $m \neq m'$ be such that $\phi(m) \neq \phi(m')$, and then pick any $\succeq$ such that $m \in \mu(\succeq)$.

Since there are no restrictions on $\mathscr{E}$, pick an extension $\succeq^* \in \mathscr{E}(\succeq)$ such that $\phi(m') \succ^* f$ for every act $f \neq \phi(m')$. But then $\phi(m') \succ^* \phi(m) \in \mu(\succeq)$, contradicting incentive compatibility.

Step 3: $\text{Range}(\phi) \subseteq D_i$ for every $i \in \{1,\ldots,k\}$.

Suppose not. Then there is some $a \in \text{Range}(\phi)$ (by step 2) and some D_j with $x, y \in D_j$ $(x \neq y)$ where $a \notin D_j$. Now pick a preference $\succeq$ where $a \succ x \succ z$ for every $z \in X \setminus \{a, x\}$, and a strict preference $\succeq'$ where $a \succ' y \succ' z$ for every $z \in X \setminus \{a, y\}$. Let $m = \mu(\succeq)$ and $m' = \mu(\succeq')$, and note that $m_j = x$ and $m'_j = y$, so $m \neq m'$. Incentive compatibility requires that $\phi(m) = a$ and $\phi(m') = a$. But incentive compatibility also requires that $\phi(m) \succ^* \phi(m')$, which is a contradiction.

Step 4: $\text{Range}(\phi) = D_i$ for every $i \in \{1,\ldots,k\}$.

Suppose not. Then there is some D_i and some $a \in D_i$ such that $a \notin \text{Range}(\phi)$. Let $\succeq$ be a preference where $a \succ z$ for every $z \neq a$, and let $m = \mu(\succeq)$. Let $b = \phi(m)$, and note that $b \in D_i$ by step 3. Now let $\succeq'$ be a strict preference where $b \succ' z$ for every $z \neq b$, and let $m' = \mu(\succeq')$. Since $m'_i = b$ and $m_i = a$, we have $m' \neq m = \mu(\succeq)$. Therefore, incentive compatibility requires that $\phi(m) \succ \phi(m')$. But incentive compatibility also requires that $\phi(m') = b$, so that $\phi(m') = \phi(m)$, a contradiction.

Step 5: $k = 1$.

Suppose not. By step 4, we have $D_1 = D_2 = \text{Range}(\phi)$. Pick any m' such that $m'_1 \neq m'_2$, and let $x = \phi(m')$. Now consider the preference $\succeq$ where $x \succ z$ for every $z \neq x$, and let $m = \mu(\succeq)$. Since $m_1 = m_2 = x$, we have that $m \neq m'$. Incentive compatibility requires that $\phi(m) \succ \phi(m')$, but also that $\phi(m) = x = \phi(m')$, a contradiction.

A.2: Proof of Lemma 1

Fix a preference $\succeq$, a truthful message $m^* \in \mu(\succeq)$, and an arbitrary message m. If $\phi(m^*)$ dominates $\phi(m)$ under $\succeq$ then, under monotonicity, $\phi(m^*) \succeq^* \phi(m)$ for any extension $\succeq^* \in \mathscr{E}(\succeq) \subseteq \mathscr{E}^{\text{mon}}(\succeq)$ (with strict orderings when $m \notin \mu(\succeq)$). Since this holds for all $\succeq$, the experiment is incentive compatible.

A.3: Proof of Lemma 2

Let $\succeq$ be a preference, and let $m^* \in \mu(\succeq)$. We claim that $\phi(m^*)$ dominates $\phi(m)$ under $\succeq$. To this end, we know that for all $\succeq^* \in \mathscr{E}(\succeq)$, we have $\phi(m^*) \succeq^* \phi(m)$. Because of our hypothesis that $\bigcap_{\succeq^* \in \mathscr{E}(\succeq)} \succeq^* = \sqsupseteq$, this implies that $\phi(m^*) \sqsupseteq \phi(m)$. Now, suppose that $m \notin \mu(\succeq)$. Then, for all i, it follows by definition that $m^*_i \succeq m_i$, and that there exists j for which $m^*_j \succ m_j$. Therefore, for all $\succeq^* \in \mathscr{E}(\succeq)$, we have $\phi(m^*) \succ^* \phi(m)$. Consequently, by our hypothesis that $\bigcap_{\succeq^* \in \mathcal{E}(\succeq)} \succeq^* = \sqsupseteq$, we know that $\phi(m) \sqsupseteq \phi(m^*)$ is false. Further, we know that $\phi(m^*) \sqsupseteq \phi(m)$ is true. Conclude that $\phi(m^*) \sqsupset \phi(m)$.

A.4: Proof of Theorem 1

The proof of the theorem follows from the following lemma:

Lemma 4. Let ϕ be an RDS mechanism, $f \in \phi(\mu(\succeq))$ and $g \in \phi(M)$.

(1) $f \sqsupseteq g$.
(2) $g \notin \phi(\mu(\succeq))$ if and only if $f \sqsupset g$.

Proof. Let $f = \phi(m^*)$ for some $m^* \in \mu(\succeq)$ and $g = \phi(m)$ for some $m \in M$. For each $i \in \{1,\ldots,k\}$, m_i^* and m_i are both elements of D_i and $m_i^* \succeq m_i$. If $g \notin \phi(\mu(\succeq))$ then $m \notin \mu(\succeq)$, and so there is some i for which $m_i^* \succ m_i$.

Since ϕ is an RDS mechanism and $f = \phi(m^*)$ for some $m^* \in \mu(\succeq)$, it must be that for each $i \in \{1,\ldots,k\}$ and $\omega \in \Omega_i$, $f(\omega) = m_i^* \succeq m_i = g(\omega)$. Thus, $f \sqsupseteq g$, proving part (1) of the lemma. If $g \notin \phi(\mu(\succeq))$ then $m \notin \mu(\succeq)$, and so there is some j where, for each $\omega \in \Omega_j$, $f(\omega) = m_j^* \succ m_j = g(\omega)$. In that case, $f \sqsupset g$. Conversely, if $f \sqsupset g$ then there is at least one state ω where $f(\omega) \succ g(\omega)$. Let i be such that $\omega \in \Omega_i$. Thus, $f(\omega) = m_i^* \succ m_i = g(\omega)$, proving that $m \notin \mu(\succeq)$ and, therefore, $g \notin \phi(\mu(\succeq))$. □

Returning to the theorem, assume incentive compatibility with respect to some $\mathcal{E}$, and pick an arbitrary $\succeq$. If $f \in \phi(\mu(\succeq))$, $g \in \phi(M)$, and $f \sqsupseteq g$, then incentive compatibility guarantees that $f \succeq^* g$ for every $\succeq^* \in \mathcal{E}(\succeq)$. (This is true regardless of ϕ, and the fact that $f \sqsupseteq g$.) Now assume $f \in \phi(\mu(\succeq))$, $g \in \phi(M)$, and $f \sqsupset g$. By part (2) of Lemma 4, $g \notin \phi(\mu(\succeq))$, so incentive compatibility guarantees that $f \succ^* g$ for all $\succeq^* \in \mathcal{E}(\succeq)$. Thus, $\mathcal{E} \subseteq \mathcal{E}^{\phi\text{mon}}$.

Conversely, assume $\mathcal{E} \subseteq \mathcal{E}^{\phi\text{mon}}$, and pick an arbitrary $\succeq$ and extension $\succeq^* \in \mathcal{E}(\succeq)$. Pick any $f \in \phi(\mu(\succeq))$ and $g \in \phi(M)$. By part (1) of Lemma 4, we have that $f \sqsupseteq g$. Since $\succeq^* \in \mathcal{E}^{\phi\text{mon}} \subseteq \mathcal{E}^{\text{mon}}$, we have $f \succeq^* g$, as needed. If, in addition, $g \notin \phi(\mu(\succeq))$, then, by part (2) of Lemma 4, $f \sqsupset g$. Here, $\succeq^* \in \mathcal{E}^{\phi\text{mon}} \subseteq \mathcal{E}^{\text{mon}}$ guarantees that $f \succ^* g$, proving that incentive compatibility holds.

A.5: Proof of Proposition 2

Consider any two acts f and g such that $f \sqsupseteq g$. Let $\{\Omega_1,\ldots,\Omega_k\}$ be the coarsest refinement of the pre-images of f and g. For each Ω_i, pick any $\omega_i \in \Omega_i$ and let $D_i = \{f(\omega_i), g(\omega_i)\}$. Consider an RDS mechanism ϕ for this $D = (D_1,\ldots,D_k)$. Let $m^* = (f(\omega_1),\ldots,f(\omega_k))$

and $m = (g(\omega_1), \ldots, g(\omega_k))$. Since $f \sqsupseteq g$, we have that $m^* \in \mu(\succeq)$. Thus, $f = \phi(m^*) \in \phi(\mu(\succeq))$ and $g \in \phi(M)$, and so, by ϕ-monotonicity, $f \succeq^* g$. If $f \sqsupset g$ then $f(\omega_i) \succ g(\omega_i)$ for at least one i, so $m \notin \mu(\succeq)$. By ϕ-monotonicity, $f \succ^* g$. Thus, monotonicity holds.

A.6: Proof of Proposition 3

Suppose that $E \in SI(\mathscr{D})$, and that E is not a singleton. Let $\{x,y\} \subseteq E$ be arbitrary. Consider two linear orders, $\succeq$ and $\succeq'$, which are identical except in their ranking of x and y (which are adjacent). They rank all elements of $X \setminus E$ above all elements of E, and they rank x and y above all elements of $E \setminus \{x,y\}$. However, $x \succ y$ and $y \succ' x$. It is clear that if there is no $D \in \mathscr{D}$ such that $\{x,y\} \subseteq D \subseteq E$, then for all $D \in \mathscr{D}$, we have $\text{dom}_{\succeq} D = \text{dom}_{\succeq'} D$, yet $\text{dom}_{\succeq} E = x \neq y = \text{dom}_{\succeq'} E$, contradicting sure identification.

Conversely, suppose that for every pair $\{x,y\} \subseteq E$, there exists $D \in \mathscr{D}$ for which $\{x,y\} \subseteq D \subseteq E$. Suppose by means of contradiction that there exist $\succeq$ and $\succeq'$ for which for all $D \in \mathscr{D}$, $\text{dom}_{\succeq} D = \text{dom}_{\succeq'} D$, but $\text{dom}_{\succeq} E \neq \text{dom}_{\succeq'} E$. Let $w = \text{dom}_{\succeq} E$ and $z = \text{dom}_{\succeq'} E$. There exists $D' \in \mathscr{D}$ for which $\{w,z\} \subseteq D' \subseteq E$. As a consequence, $w = \text{dom}_{\succeq} D'$ and $z = \text{dom}_{\succeq'} D'$, contradicting the fact that $\text{dom}_{\succeq} D = \text{dom}_{\succeq'} D$ for all $D \in \mathscr{D}$.

A.7: Proof of Theorem 2

We start by showing that any RSS mechanism that satisfies conditions (1) and (2) is incentive compatible (with respect to $\mathscr{E}^{\text{mon}}$). Let $\succeq$ be arbitrary, $m^* = \mu(\succeq)$ and let $\succeq^*$ be some (monotonic) extension of $\succeq$. We claim that $\phi(m^*) \succeq^* \phi(m')$ for any $m' \neq m^*$. This follows since, for each ω, $\phi(m^*)(\omega) = \text{dom}_{m^*}(P^{\phi}(\omega))$ and $\phi(m')(\omega) \in P^{\phi}(\omega)$, so $\phi(m^*)(\omega) \succeq \phi(m')(\omega)$. Since $m' \neq \mu(\succeq)$, we must also show that there exists $\omega \in \Omega$ for which $\phi(m^*)(\omega) \succ \phi(m')(\omega)$. Suppose not, so that $\phi(m^*)(\omega) \sim \phi(m')(\omega)$ at each ω. Because $\succeq$ is a linear order, this implies that $\phi(m^*) = \phi(m')$. Recalling condition (2) of the hypothesis, this implies that $m' \in M_R$, so there exists $\succeq'$ for which $m' = \mu(\succeq')$. Since $\phi(m^*) = \phi(m')$, both acts pick the same elements from every $P^{\phi}(\omega)$. Condition (1) requires $D_i \in SI(\mathscr{P}^{\phi})$ for every i, so that $\mu_i(\succeq) = \mu_i(\succeq')$ for every i. But $\mu(\succeq) = \mu(\succeq')$ contradicts $m^* \neq m'$.

Conversely, let ϕ be an incentive compatible mechanism for $(D_1, \ldots, D_k)$. Recall that, for each $\omega \in \Omega$, $P^{\phi}(\omega) = \phi(M)(\omega)$. Let $m^* \in M_R$, and let $\succeq$ such that $m^* = \mu(\succeq)$. By incentive compatibility (recall Lemma 2), it follows that for all $m \in M$, we have $\phi(\mu(\succeq)) \sqsupseteq \phi(m)$. In particular, this implies that, for all $\omega \in \Omega$, $\phi(\mu(\succeq))(\omega) \succeq \phi(m)(\omega)$ (by definition of $\sqsupseteq$), or $\phi(m^*)(\omega) \succeq \phi(m)(\omega)$. That is, $\phi(m^*)(\omega) \succeq y$ for all $y \in P^{\phi}(\omega)$. Since $\succeq$ was arbitrary, this establishes both that $P^{\phi}(\omega) \in SI(\mathscr{D})$, and that $\phi(m)(\omega) = \mathrm{dom}_m(P^{\phi}(\omega))$ whenever $m \in M_R$. Hence, ϕ is an RSS.

We claim now that for all i, $D_i \in SI(\mathscr{P}^{\phi})$. If not, then by definition, there exists D_i, and preferences $\succeq, \succeq'$ for which for all $\omega \in \Omega$, $\mathrm{dom}_{\succeq} P^{\phi}(\omega) = \mathrm{dom}_{\succeq'} P^{\phi}(\omega)$, but for which $\mathrm{dom}_{\succeq} D_i \neq \mathrm{dom}_{\succeq'} D_i$. Hence, $\mu(\succeq) \neq \mu(\succeq')$. Since ϕ is an RSS mechanism, for all $\omega \in \Omega$, $\phi(\mu(\succeq))(\omega) = \mathrm{dom}_{\succeq} P^{\phi}(\omega) = \mathrm{dom}_{\succeq'} P^{\phi}(\omega) = \phi(\mu(\succeq'))(\omega)$. Consequently, $\phi(\mu(\succeq)) = \phi(\mu(\succeq'))$, but $\mu(\succeq) \neq \mu(\succeq')$. In particular, since $\mu(\succeq)$ and $\mu(\succeq')$ are each single-valued, incentive compatibility implies that there exists $\omega \in \Omega$ for which $\phi(\mu(\succeq))(\omega) \succ \phi(\mu(\succeq'))(\omega)$, a contradiction.

Finally, suppose that there are $m \in M_R$ and $m' \in M_{NR}$ such that $\phi(m) = \phi(m')$. Let $\succeq$ be such that $\mu(\succeq) = m$. Incentive compatibility requires that $\phi(m) \sqsupset \phi(m')$ with respect to $\succeq$, which contradicts $\phi(m) = \phi(m')$.

A.8: Proof of Corollary 2

We start with the following lemma.

Lemma 5. Let ϕ be an RSS mechanism that satisfies condition (2) of Theorem 2. Then ϕ is incentive compatible if and only if $\mathrm{NRed}(\mathscr{D}) \subseteq \mathscr{P}^{\phi}$.

Proof. The proof follows immediately from Theorem 2, since all we need to show is that condition (1) of that theorem is equivalent to $\mathrm{NRed}(\mathscr{D}) \subseteq \mathscr{P}^{\phi}$.

Suppose first that any decision problem is surely identified from $\mathscr{P}^{\phi}$, and let D_i be non-redundant. Assume by contradiction that $D_i \notin \mathscr{P}^{\phi}$. By Proposition 3, for each pair $\{x, y\} \subseteq D_i$ there is some $\{x, y\} \subseteq P^{\phi}(\omega_{(x,y)}) \subsetneq D_i$. Now, as each $P^{\phi}(\omega_{(x,y)}) \in SI(\mathscr{D})$, and since each $P^{\phi}(\omega_{(x,y)}) \subsetneq D_i$, it follows by Proposition 3 that $P^{\phi}(\omega_{(x,y)}) \in SI(\mathscr{D} \setminus \{D_i\})$. This tells us that there exists $D_{(x,y)}$ for which $\{x, y\} \subseteq D_{(x,y)} \subseteq P^{\phi}(\omega_{(x,y)})$. Further, we know that $\{x, y\} \subseteq D_{(x,y)} \subsetneq D_i$. By Proposition 3, it follows that D_i is surely identified by the $D_{(x,y)}$, contradicting the fact that D_i is non-redundant.

Conversely, suppose that $\mathrm{NRed}(\mathscr{D}) \subseteq \mathscr{P}^{\phi}$ and let D_i be some decision problem. If D_i is non-redundant then obviously it is surely identified from the payment sets, so assume D_i is redundant. Then by Proposition 3 for every $\{x, y\} \subseteq D_i$ there is $D_{(x,y)}$ such that $\{x, y\} \subseteq D_{(x,y)} \subsetneq D_i$. For each $\{x, y\}$ choose a minimal (with respect to inclusion) set $D_{(x,y)}$ with the above property. Then each $D_{(x,y)}$ is non-redundant and this collection of sets surely identifies D_i. It follows that D_i is surely identified from the payment sets, so ϕ is incentive compatible. □

We are now ready to prove the corollary. First, suppose that $SI(\mathscr{D})\setminus\{\{x\}_{x\in X}\} = \text{NRed}(\mathscr{D})$. We know from Theorem 2 that any incentive compatible mechanism ϕ is an RSS mechanism, and from the above lemma that any non-redundant set must be a payment set at some state. It follows from the assumption that $\mathscr{D} = \text{NRed}(\mathscr{D})$, so every decision problem is a payment set at some state. Furthermore, since any payment set is in $SI(\mathscr{D})$, it follows that whenever $\mathscr{P}^{\phi}(\omega)$ is not a singleton it is in $\mathscr{D}$. This proves that ϕ coincides with an RDS mechanism when restricted to M_R and to states where the payment sets of ϕ are not singletons.

Now, suppose that any incentive compatible mechanism ϕ coincides with an RDS mechanism restricted to M_R and to states where the payment sets of ϕ are not singletons. If $\text{NRed}(\mathscr{D}) \subsetneq \mathscr{D}$ then, by the above lemma, there are incentive compatible mechanisms in which $\mathscr{P}^{\phi} \subsetneq \mathscr{D}$, contradicting the assumption. Thus, we must have $\text{NRed}(\mathscr{D}) = \mathscr{D}$. Similarly, we must also have $SI(\mathscr{D}) \setminus \{\{x\}_{x\in X}\} = \mathscr{D}$, and the result follows.

A.9: Proof of Corollary 3

We show that the condition from Corollary 2 is satisfied. To this end, we will show that all nontrivial elements of $SI(\mathscr{D})$ are elements of $\mathscr{D}$, and that all elements of $\mathscr{D}$ are non-redundant.

For the first claim, suppose by means of contradiction that $E \in SI(\mathscr{D})$, but that $E \notin \mathscr{D}$. We use Proposition 3 without explicit reference. Let $x, y \in E$. There is $D_{(x,y)} \in \mathscr{D}$ for which $\{x, y\} \subseteq D_{(x,y)} \subsetneq E$. There exists $z \in E\setminus D_{(x,y)}$. Let $D_{(x,z)} \in \mathscr{D}$ be such that $\{x,z\} \subseteq D_{(x,z)}$. If $y \in D_{(x,z)}$, then an irrational message exists (namely, announcing x from E and y from $D_{(x,z)}$. If $y \notin D_{(x,z)}$, then let $D_{(y,z)} \in \mathscr{D}$ for which $\{y,z\} \subseteq D_{(y,z)}$. Then, an irrational message exists, namely, announcing x from E, y from $D_{(y,z)}$ and z from $D_{(x,z)}$.

For the second claim, suppose that there is a redundant D_i. Then, similarly to the preceding paragraph, we can construct an irrational message from the elements of $\mathscr{D}\setminus D_i$.

Now, since $M_{NR} = \varnothing$, we do not need to add the qualification that there is an agreeing RDS mechanism on M_R (since $M = M_R$).

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