Gains from Trade

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Abstract

In a market design context, we ask whether there exists a system of transfers and regulations whereby gains from trade can always be realized under trade liberalization. In a model with a sufficiently rich class of admissible preferences, we show that the answer is negative. This holds in particular when starting from a Walrasian mechanism, but more importantly, holds for any mechanism which always recommends Pareto efficient allocations and only takes preferences over traded commodities into account. The only exception is a dictatorial mechanism, whereby one agent realizes all the gains from trade.

1 Introduction

A classical rationale for markets is that they allow gains from trade to be realized; at the very least, no agent can be made worse off than her initial holding. However, this basic comparative static only holds generally when starting from autarky. If a group of agents trade some goods on the market, but others are untraded, opening markets in the untraded goods can potentially hurt some of the agents. The intuition for this is simple: opening trade in new goods can alter the equilibrium price of already traded goods to accommodate the potential tradeoffs for newly traded goods.

In the international trade literature, this is known as a negative terms-of-trade effect (see Krugman, Obstfeld and Melitz [6] for example). A related phenomenon occurs in

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the context of financially incomplete markets. Hart [4] offers an example establishing that opening a market in new securities result in a Pareto loss. Also, Elul [2] and Cass and Citanna [1] have shown that such worsening is generic.

A very basic question remains. While unregulated markets do not in general produce gains from trade except in the special case of autarky, there may be room for transfers or subsidies or regulations which allow such a result to be restored more generally. To this end, our question does not take the competitive market mechanism in the Walrasian sense as given. We ask: Is it possible to allocate resources, allowing redistribution of income or resources and any other compensation or any price regulation, so that that opening trade in new goods never makes anybody worse off?

Somewhat surprisingly, we show that the answer is generally negative. To qualify this statement, we first ask what we demand of our mechanism. First, we ask that our mechanism always respect weak Pareto efficiency. Secondly, we ask that our mechanism be sufficiently *decentralized*, in that it only take into account preferences and endowments of traded commodities. Any mechanism not satisfying this property would require extreme bureaucratic involvement on the part of a social planner, requiring sophisticated knowledge of preferences over untraded commodities. The Walrasian mechanism, for example, satisfies these two properties. Finally, we ask that nobody be made worse off when opening markets to trade in new goods.

The idea of alternative market mechanisms is largely a philosophical one. The state of the world economy is one in which, to a very rough first approximation, traded goods are traded via some version of a price mechanism. From a practical standpoint, therefore, the relevant question seems to be whether or not, *taking the Walrasian mechanism's allocation as given*, it is possible to open trade in new goods without hurting somebody. We establish that this is generically impossible.

Related literature

Mathematically, our result is probably closest to Fleurbaey and Tadenuma [3], who establish an Arrovian aggregation result using an independence condition analogous to our decentralization condition mentioned above. Aside from the obvious difference that their exercise involves ranking bundles, they also make a (somewhat unusual) assumption that when ranking consumption of traded commodities, the consumption of all other commodities for all agents is zero. This assumption is useful for an extension result used in their proof. Because we make no such assumption, we cannot rely on the extension result constructed in their paper and must proceed in a different route.

Conceptually; however, our result is related to several results in the literature in social choice in exchange economies, for example Moulin and Thomson [7]. A major theme of this literature relates to whether everybody can benefit systematically when the set of available objects increases somehow. The aforementioned result establishes that, in an exchange economy environment without endowments, it is very hard for each agent to benefit when more of each commodity is introduced. Our result follows this theme by considering the introduction of new commodities, rather than introducing more of existing commodities.

2 Model and axioms

We consider a *potential commodities*, indexed by $\mathbb{N}$. The family of finite subsets of $\mathbb{N}$ is denoted by $\mathcal{X}$. An *economy* consists of a set of *physically present* commodities $X \in \mathcal{X}$, an endowment $\omega_i \in \mathbb{R}^X_+$ and a preference relation $\succsim_i$ over $\mathbb{R}^X_+$ for each $i \in I$, and a set of *tradable* commodities $T \in \mathcal{X}$ with $T \subset X$.

Given $X \in \mathcal{X}$, let $\mathcal{R}_X$ be a domain of preferences over $\mathbb{R}^X_+$, where all elements of $\mathcal{R}_X$ are convex and strongly monotone. Let $\{\mathcal{R}_X\}_{X\in\mathcal{X}}$ denote the family of preference domains.

The set of economies is written $\mathcal{E}$, where

$$\mathcal{E} = \left\{(X, \omega, \succsim, T) : X, T \in \mathcal{X},\ \omega \in \mathbb{R}^{I\times X}_+,\ \succsim \in \mathcal{R}^I_X,\ T \subset X\right\}.$$

For a given economy $e \in \mathcal{E}$, $X(e)$ denotes the set of commodities physically present in that economy, $\omega(e) = (\omega_1(e), \cdots, \omega_{|I|}(e)) \in \mathbb{R}^{I\times X}_+$ denotes the endowments, $\succsim(e) = (\succsim_1(e), \cdots, \succsim_{|I|}(e)) \in \mathcal{R}^I_{X(e)}$ is the preference profile, and $T(e)$ denotes the set of tradable commodities. Further, let $F(e) \subset \mathbb{R}^{I\times X(e)}_+$ denotes the set of feasible allocations in e, defined by

$$F(e) = \left\{ x \in \mathbb{R}^{I\times X(e)}_+ : \begin{array}{c} \sum_{i\in I} x_{ik} \leq \sum_{i\in I} \omega_{ik}(e),\ \forall k \in T(e) \\ x_{ik} = \omega_{ik},\ \forall i \in I,\ \forall k \in X(e) \setminus T(e) \end{array} \right\}.$$

A *mechanism* is a function φ carrying each economy $e \in \mathcal{E}$ into an element of $F(e)$. A mechanism specifies how trade in any given economy should be undertaken. The use of an abstract mechanism allows us to study the properties we wish our allocations to satisfy.

We list here our properties for mechanisms. The first states that, for any given economy, it should be impossible to reallocate tradable resources in a fashion that makes everybody strictly better off.

Axiom 1 (**Weak Pareto**): For all $e \in \mathcal{E}$, there is no $x \in F(e)$ such that $x_i \succ_i (e)\varphi_i(e)$ for $i \in I$.

The second condition is our motivating criterion: when opening up trade in new commodities, nobody should be hurt.

Axiom 2 (**No Loss from Trade**): For all $e, e' \in \mathcal{E}$ with $X(e) = X(e')$, $\omega(e) = \omega(e')$, $\succsim (e) = \succsim (e')$, if $T(e) \subset T(e')$ then

$$\varphi_i(e') \succsim_i (e)\varphi_i(e)$$

for $i \in I$.

Note that no loss from trade implies the following individual rationality axiom:

Axiom 3 (**Individual Rationality**): For all $e \in \mathcal{E}$,

$$\varphi_i(e) \succsim_i (e)\omega_i(e)$$

for $i \in I$.

Finally, we specify our decentralization condition. Formally this is an independence condition, specifying that only the preferences over tradable commodities should be taken into account by the mechanism. Any mechanism not satisfying this property will necessarily be extremely complicated.

Axiom 4 (**Independence of Untraded Commodities**): For all $e, e' \in \mathcal{E}$ with $T(e) = T(e') \equiv T$, $\omega_T(e) = \omega_T(e')$ and if

$$\begin{aligned} & (x, \omega_{i,X(e)\setminus T}) \succsim_i (e)(y, \omega_{i,X(e)\setminus T}) \\ \iff & (x, \omega_{i,X(e')\setminus T}) \succsim_i (e')(y, \omega_{i,X(e')\setminus T}) \end{aligned}$$

for all $i \in I$ and $x, y \in \mathbb{R}^T_+$, then

$$\varphi_T(e') = \varphi_T(e).$$

Examples of mechanisms satisfying all but one of the properties follow.

Example 1 **No-trade solution** which gives $\varphi(e) = \omega(e)$ for all $e \in \mathcal{E}$ satisfies No Loss from Trade, Independence of Untraded Commodities but violates Weak Pareto.

Example 2 **Monotone path solution** is defined as follows. For all $X \in \mathcal{X}$ and $R \in \mathcal{R}_X^I$, fix a profile of utility representations $u = (u_i)_{i \in I}$.

For all $e \in \mathcal{E}$, define

$$\varphi(e) \in \arg \max_{x \in F(e)} \min_{i \in I} u_i(x_i),$$

in which the way of selection when multiplicity occurs is arbitrary.

This satisfies Weak Pareto, No Loss from Trade but violates Independence of Untraded Commodities.

Example 3 Consider any selection of **Walrasian solution**, in which the way of selection when multiplicity occurs depends only on preference induced over the tradable commodities.

This satisfies Weak Pareto, Independence of Untraded Commodities but violates No Loss from Trade.

Let us conclude this section by showing that impossibility follows from a simple example when weak monotonicity of preference is allowed.

Example 4 Consider the two-commodity economy, $X = \{1, 2\}$ whereby each agent has preference represented by the utility function $u(x, y) = \min\{x, y\}$. Individual A has an endowment of $(1, 0)$ and B an endowment of $(0, 1)$. Consider now $e|_{\{1\}}$. Clearly, for any $x \in \mathbb{R}$, $(x, 0) \sim_A (1, 0)$. In particular, the only Pareto efficient allocation for this economy consists in giving all of commodity one to B: $(0, 0), (1, 1)$. Likewise, the only Pareto efficient allocation for $e|_{\{2\}}$ is $(1, 1), (0, 0)$. Now, it is clear that No Loss from Trade cannot be satisfied.

3 A dictatorship characterization

Let

$$
\begin{aligned}
P(x_i, \succsim_i) &= \{z_i \in \mathbb{R}^X : z_i \succ_i x_i\} \\
R(x_i, \succsim_i) &= \{z_i \in \mathbb{R}^X : z_i \succsim_i x_i\} \\
P_C(x_i, \succsim_i, \omega_i) &= \{z_{iC} \in \mathbb{R}^C : (z_{iC}, \omega_{iD}) \succ_i x_i\} \\
R_C(x_i, \succsim_i, \omega_i) &= \{z_{iC} \in \mathbb{R}^C : (z_{iC}, \omega_{iD}) \succsim_i x_i\}
\end{aligned}
$$

for each i.

Condition 1 (**Richness**): (1) For any $X \in \mathcal{X}$, any $\succsim \in \mathcal{R}_X^I$, $\omega \in \mathbb{R}_+^{I\times X}$ and any partition of X denoted by $\{C, D\}$, there exists $\succsim^* \in \mathcal{R}_X^I$ such that for each $i \in I$ and for all $x_{iC}, y_{iC} \in \mathbb{R}_+^C$, $x_{iD}, y_{iD} \in \mathbb{R}_+^D$ it holds

$$(x_{iC}, \omega_{iD}) \succsim_i (y_{iC}, \omega_{iD}) \iff (x_{iC}, \omega_{iD}) \succsim_i^* (y_{iC}, \omega_{iD})$$

$$(\omega_{iC}, x_{iD},) \succsim_i (\omega_{iC}, y_{iD}) \iff (\omega_{iC}, x_{iD},) \succsim_i^* (\omega_{iC}, y_{iD}).$$

Moreover, for any fixed $x \in \mathbb{R}_+^{I\times X}$ such that

$$
\begin{aligned}
\sum_{i\in I} \omega_{iC} &\notin \sum_{i\in I} R_C(x_i, \succsim_i, \omega_i) \\
\sum_{i\in I} \omega_{iD} &\notin \sum_{i\in I} R_D(x_i, \succsim_i, \omega_i)
\end{aligned}
$$

and

$$
\begin{aligned}
\omega_{iC} &\notin R_C(x_i, \succsim_i, \omega_i) \\
\omega_{iD} &\notin R_D(x_i, \succsim_i, \omega_i)
\end{aligned}
$$

for all i, one can take $\succsim^* \in \mathcal{R}_X^I$ so that

$$\sum_{i\in I} \omega_i \notin \sum_{i\in I} R(x_i, \succsim_i^*)$$

(2) For any $X, X' \in \mathcal{X}$ with $X \cap X' = \emptyset$, for any $\succsim \in \mathcal{R}_X^I$, $\succsim' \in \mathcal{R}_{X'}^I$ and any $\omega \in \mathbb{R}_+^X$, $\omega' \in \mathbb{R}_+^{X'}$, there exists $\succsim^* \in \mathcal{R}_{X\cup X'}^I$ such that for each $\in I$ and for all $y_i, z_i \in \mathbb{R}_+^X$ and $y_i', z_i' \in \mathbb{R}_+^{X'}$ it holds

$$y_i \succsim_i z_i \iff (y_i, \omega_i') \succsim_i^* (z_i, \omega_i')$$

$$y'_i \succsim'_i z'_i \iff (\omega_i, y'_i) \succsim^*_i (\omega_i, z'_i)$$

Moreover, for each $i \in I$ for any fixed $x_i \in \mathbb{R}^X_+$, $x'_i \in \mathbb{R}^{X'}_+$ with $x_i \succ_i \omega_i$ and $x'_i \prec'_i \omega'_i$ (resp. $x_i \prec_i \omega_i$ and $x'_i \succ'_i \omega'_i$), one can take $\succsim^*_i \in \mathcal{R}_{X \cup X'}$ so that any of $(x_i, \omega'_i) \succ^*_i (\omega_i, x'_i)$ or $(x_i, \omega'_i) \prec^*_i (\omega_i, x'_i)$ holds.

The condition states that: (1) given an economy, an allocation and any two subeconomies such that any better allocation is not attainable either by the entire society or by any individual in either of the subeconomies, there is a preference profile which agrees on the original one on each of the subeconomies and any better allocation is not attainable either by the entire society in the entire economy; and (2) given any two economies there is a preference profile for the larger economy obtained by merging the two economies which agrees the original one on each of the economies and allows flexible rankings between any fixed allocations in the larger economy.

Although it is an open question as to whether there exists a particular preference domain which satisfies the condition, we think it is intuitive in the sense that (1) says one can take a preference profile which agrees with the original one on each of the subeconomies but more "demanding" in the entire economy, and (2) says one can "glue" indifference surfaces in two economies so as to allow for flexible rankings in the larger economy. We conjecture that the domain of all strictly monotone and strictly convex continuous preferences satisfies the condition.

Definition 1 A solution is **dictatorial** if there exists $i \in I$ such that for all $e \in \mathcal{E}$ it holds

$$\varphi_i(e) \succsim_i (e) x_i$$

for all $x \in F(e)$ satisfying $x_j \succsim_j (e) \omega_j(e)$ for $j \neq i$.

Theorem 1 Assume Richness. Then the only solution which satisfies Weak Pareto, No Loss from Trade and Independence of Untraded Commodities is dictatorship.

First we prove a lemma saying that the outcomes of the rule applied to two subeconomies with mutually disjoint sets of tradable commodities must be Pareto ranked. Given $e \in \mathcal{E}$ and $C \subset T(e)$, let $e|_C$ denote an economy such which $X(e|_C) = X(e)$, $\omega(e) = \omega(e|_C)$, $\succsim (e|_C) = \succsim (e)$ and $T(e|_C) = C$.

Lemma 1 For all e with $|X(e)| \geqq 4$ and $T(e) = X(e)$ and any partition $\{C, D\}$ of $X(e)$ with $|C|, |D| \geqq 2$, $\varphi(e|_C)$ and $\varphi(e|_D)$ are Pareto ranked.

Proof. Let

$$\begin{aligned} I_1 &= \{i \in I : \varphi_i(e|_C) \succ_i (e)\varphi_i(e|_D)\} \\ I_2 &= \{i \in I : \varphi_i(e|_C) \prec_i (e)\varphi_i(e|_D)\} \\ I_3 &= \{i \in I : \varphi_i(e|_C) \sim_i (e)\varphi_i(e|_D)\} \end{aligned}$$

and suppose $I_1, I_2 \neq \emptyset$.

By Weak Pareto we have

$$\sum_{i \in I} \omega_{iC}(e) \notin \sum_{i \in I} P_C(\varphi_i(e|_C), \succsim_i (e), \omega_i(e))$$

For each $i \in I_2$, by assumption that $\varphi_i(e|_C) \prec_i (e)\varphi_i(e|_D)$, it follows $R_C(\varphi_i(e|_D), \succsim_i (e), \omega_i(e)) \subsetneqq P_C(\varphi_i(e|_C), \succsim_i (e), \omega_i(e))$. Therefore we have

$$\sum_{i \in I} \omega_{iC}(e) \notin \sum_{i \in I_1 \cup I_3} R_C(\varphi_i(e|_C), \succsim_i (e), \omega_i(e)) + \sum_{i \in I_2} R_C(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$$

By Weak Pareto we have

$$\sum_{i \in I} \omega_{iD}(e) \notin \sum_{i \in I} P_D(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$$

For each $i \in I_1$, by assumption that $\varphi_i(e|_C) \succ_i (e)\varphi_i(e|_D)$, it follows $R_D(\varphi_i(e|_C), \succsim_i (e), \omega_i(e)) \subsetneqq P_D(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$. Therefore we have

$$\sum_{i \in I} \omega_{iD}(e) \notin \sum_{i \in I_1 \cup I_3} R_D(\varphi_i(e|_C), \succsim_i (e), \omega_i(e)) + \sum_{i \in I_2} R_D(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$$

By the richness condition we can take e^* such that $X(e^*) = X(e)$, $\omega(e^*) = \omega(e)$ and $T(e^*) = X(e^*)$ such that for each $i \in I$ and for all $x_{iC}, y_{iC} \in \mathbb{R}^C_+$, $x_{iD}, y_{iD} \in \mathbb{R}^D_+$ it holds

$$(x_{iC}, \omega_{iD}) \succsim_i (y_{iC}, \omega_{iD}) \iff (x_{iC}, \omega_{iD}) \succsim_i^* (y_{iC}, \omega_{iD})$$

$$(\omega_{iC}, x_{iD},) \succsim_i (\omega_{iC}, y_{iD}) \iff (\omega_{iC}, x_{iD},) \succsim_i^* (\omega_{iC}, y_{iD}),$$

and

$$\sum_{i \in I} \omega_i(e^*) \quad \notin \quad \sum_{i \in I_1 \cup I_3} R(\varphi_i(e|_C), \succsim_i (e^*)) + \sum_{i \in I_2} R(\varphi_i(e|_D), \succsim_i (e^*))$$

Since $\varphi(e^*|_C) = \varphi(e|_C)$ and $\varphi(e^*|_D) = \varphi(e|_D)$, No Loss from Trade requires

$$\begin{aligned}\sum_{i \in I} \varphi_i(e^*) &\in \sum_{i \in I_1 \cup I_3} R(\varphi_i(e^*|_C), \succsim_i (e^*)) + \sum_{i \in I_2} R(\varphi_i(e^*|_D), \succsim_i (e^*)) \\ &= \sum_{i \in I_1 \cup I_3} R(\varphi_i(e|_C), \succsim_i (e^*)) + \sum_{i \in I_2} R(\varphi_i(e|_D), \succsim_i (e^*)),\end{aligned}$$

which is a contradiction to $\sum_{i \in I} \varphi_i(e^*) = \sum_{i \in I} \omega_i(e^*)$. ■

Proof of the Theorem. Because of Independence of Untraded Commodities it is without loss of generality to assume $T(e) = X(e)$.

Let e be any economy with $|X(e)| \geqq 2$. Suppose there exists $i, j \in I$ such that $\varphi_i(e) \succ_i (e)\omega_j(e)$ and $\varphi_j(e) \succ_j (e)\omega_j(e)$.

Let e' be any economy with $|X(e')| \geqq 2$ and $X(e) \cap X(e') = \emptyset$ in which there is an allocation with strict gain from trades, that is, it is possible to make a strict Pareto improvement upon $\omega(e')$. By the individual rationality condition we have $\varphi_i(e') \succsim_i (e')\omega_i(e')$ and $\varphi_j(e') \succsim_j (e')\omega_j(e')$.

Case 1: Suppose $\varphi_i(e') \succ_i (e')\omega_i(e')$ and $\varphi_j(e') \succ_j (e')\omega_j(e')$.

Then we can take an economy e^* such that $X(e^*) = X(e) \cup X(e')$, and $\succsim (e^*)$ satisfies $\succsim (e^*)|_{X(e)} = \succsim (e)$ and $\succsim (e^*)|_{X(e')} = \succsim (e')$, and

$$(\varphi_i(e), \omega_i(e')) \succ_i (e^*)(\omega_i(e), \varphi_i(e'))$$

$$(\varphi_j(e), \omega_j(e')) \prec_j (e^*)(\omega_j(e), \varphi_j(e')).$$

By Independence of Untraded Commodities, this is equivalent to

$$\varphi_i(e^*|_{X(e)}) \succ_i (e^*)\varphi_i(e^*|_{X(e')})$$

and

$$\varphi_j(e^*|_{X(e)}) \prec_j (e^*)\varphi_j(e^*|_{X(e')})$$

However, this is a contradiction to the previous lemma.

Case 2: Suppose $\varphi_i(e') \succ_i (e')\omega_i(e')$ and $\varphi_j(e') \sim_j (e')\omega_j(e')$.

Then we can take an economy e^* such that $X(e^*) = X(e) \cup X(e')$, and $\succsim (e^*)$ satisfies $\succsim (e^*)|_{X(e)} = \succsim (e)$ and $\succsim (e^*)|_{X(e')} = \succsim (e')$, and

$$(\varphi_i(e), \omega_i(e')) \prec_i (e^*)(\omega_i(e), \varphi_i(e'))$$

$$(\varphi_j(e), \omega_j(e')) \succ_j (e^*)(\omega_j(e), \varphi_j(e')).$$

By Independence of Untraded Commodities, this is equivalent to

$$\varphi_i(e^*|_{X(e)}) \prec_i (e^*)\varphi_i(e^*|_{X(e')})$$

and

$$\varphi_j(e^*|_{X(e)}) \succ_j (e^*)\varphi_j(e^*|_{X(e')})$$

However, this is a contradiction to the above lemma.

Case 3: Suppose $\varphi_i(e') \sim_i (e')\omega_i(e')$ and $\varphi_j(e') \succ_j (e')\omega_j(e')$. Then we can follow the argument similar to Case 2.

Case 4: Suppose $\varphi_i(e') \sim_i (e')\omega_i(e')$ and $\varphi_j(e') \sim_j (e')\omega_j(e')$. Then by Weak Pareto there exists $k \neq i, j$ such that $\varphi_k(e') \succ_k (e')\omega_k(e')$. By the individual rationality condition it holds $\varphi_k(e) \succsim_k (e)\omega_k(e)$. Then we can follow the argument similar to one of the above cases.

Thus, without loss of generality let's say we have $\varphi_1(e) \succ_1 (e)\omega_1(e)$ and $\varphi_i(e) \sim_i (e)\omega_i(e)$ for all $i \neq 1$.

To show that the dictator is always the same, suppose without loss of generality that $\varphi_1(e') \sim_1 (e')\omega_1(e')$ and $\varphi_2(e') \succ_2 (e')\omega_2(e')$ for any economy e' with $|X(e')| \geqq 2$ and $X(e) \cap X(e') = \emptyset$.

Then we can take an economy e^* such that $X(e^*) = X(e) \cup X(e')$, and $\succsim (e^*)$ satisfies $\succsim (e^*)|_{X(e)} = \succsim (e)$ and $\succsim (e^*)|_{X(e')} = \succsim (e')$, and

$$(\varphi_1(e), \omega_1(e')) \succ_1 (e^*)(\omega_1(e), \varphi_1(e'))$$

$$(\varphi_2(e), \omega_2(e')) \prec_2 (e^*)(\omega_2(e), \varphi_2(e')).$$

However, this is a contradiction to the above lemma. Thus we have to have $\varphi_1(e') \succ_1 (e')\omega_1(e')$ and $\varphi_2(e') \sim_2 (e')\omega_2(e')$.

Now for all e' with $|X(e')| \geqq 2$, by taking e'' with $|X(e'')| \geqq 2$, $X(e) \cap X(e'') = \emptyset$ and $X(e') \cap X(e'') = \emptyset$ and applying the above result there we obtain $\varphi_1(e') \succ_1 (e')\omega_1(e')$ and $\varphi_2(e') \sim_2 (e')\omega_2(e')$.

■

4 Impossibility of opening markets without hurting anybody

The relevant question for practical market design relates to the status quo. The status quo in the real world takes the form, to a very rough approximation, of a Walrasian mechanism over traded goods. Thus, the important question to answer from a practical standpoint is whether it is possible that opening trade to more goods can be beneficial for all participants, when *starting* from the Walrasian mechanism over traded goods.

In fact, we answer this condition in the negative. There is generally no system of transfers, taxes, subsidies or price regulation that could be used, when starting from a Walrasian allocation, that would make everybody better. off.

Here we assume that the preference domain $\{\mathcal{R}_X\}$ consists of all strictly convex and strongly monotone preferences. Let W denote the Walrasian correspondence.

Definition 2 Fix any $C, D \in \mathcal{X}$ with $C \cap D \neq \emptyset$ and $|C|, |D| \geqq 2$. Say that φ satisfies Walrasian Selection for $\{C, D\}$ if $\varphi(e) \in W(e)$ whenever $X(e) = C \cup D$ and $T(e) \in \{C, D\}$.

Theorem 2 Fix any $C, D \in \mathcal{X}$ with $C \cap D \neq \emptyset$ and $|C|, |D| \geqq 2$. Then there is no allocation rule which satisfies No Losses from Trade and Walrasian Selection for $\{C, D\}$.

For the proof we establish the following lemma.

Lemma 2 Suppose φ satisfies Efficiency and No Loss from Trade. For any e with $|X(e)| \geqq 4$ and any partition $\{C, D\}$ of $X(e)$ with $|C|, |D| \geqq 2$, suppose that $\varphi(e|_C) \in W(e|_C)$, $\varphi(e|_D) \in W(e|_D)$ and $|W(e|_C)| = |W(e|_D)| = 1$.
Then $\varphi(e|_C)$ and $\varphi(e|_D)$ are Pareto-ranked.

Proof. Let

$$
\begin{aligned}
I_1 &= \{i \in I : \varphi_i(e|_C) \succ_i (e)\varphi_i(e|_D)\} \\
I_2 &= \{i \in I : \varphi_i(e|_C) \prec_i (e)\varphi_i(e|_D)\} \\
I_3 &= \{i \in I : \varphi_i(e|_C) \sim_i (e)\varphi_i(e|_D)\}
\end{aligned}
$$

and suppose $I_1, I_2 \neq \emptyset$.

Let p_C be the price vector corresponding to $W(e|_C)$.

For each $i \in I_2$, by assumption that $\varphi_i(e|_C) \prec_i (e)\varphi_i(e|_D)$, it follows $R_C(\varphi_i(e|_D), \succsim_i (e), \omega_i(e)) \subsetneqq P_C(\varphi_i(e|_C), \succsim_i (e), \omega_i(e))$. Therefore p_C separates strictly between $\sum_{i\in I}\omega_{iC}$ and

$$\sum_{i\in I_1\cup I_3} R_C(\varphi_i(e|_C), \succsim_i (e), \omega_i(e)) + \sum_{i\in I_2} R_C(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$$

Let p_D be the price vector corresponding to $W(e|_D)$.

For each $i \in I_1$, by assumption that $\varphi_i(e|_C) \succ_i (e)\varphi_i(e|_D)$, it follows $R_D(\varphi_i(e|_C), \succsim_i (e), \omega_i(e)) \subsetneqq P_D(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$. Therefore p_D strictly separates between $\sum_{i\in I}\omega_{iD}$ and

$$\sum_{i\in I_1\cup I_3} R_D(\varphi_i(e|_C), \succsim_i (e), \omega_i(e)) + \sum_{i\in I_2} R_D(\varphi_i(e|_D), \succsim_i (e), \omega_i(e))$$

Now consider e^* with $X(e^*) = X(e)$ and $\omega(e^*) = \omega(e)$ such that $\succsim (e^*)$ is represented in the form

$$u_i(x_i|e^*) = \max_{(1-\varepsilon)V(u)+\varepsilon W(u)\ni x} u,$$

where

$$V(u) = co\left(\{(z_{iC}, \omega_{iD}) : u_i(z_{iC}, \omega_{iD}|e) \geqq u\} \cup \{(\omega_{iC}, z_{iD}) : u_i(\omega_{iC}, z_{iD}|e) \geqq u\}\right)$$

and

$$W(u) = \{z : u_i(z|e) \geqq u\}.$$

Note that $\succsim (e^*)$ is strictly convex and strictly monotone for $\varepsilon > 0$.

Then for each $i \in I$ and for all $x_{iC}, y_{iC} \in \mathbb{R}^C_+$, $x_{iD}, y_{iD} \in \mathbb{R}^D_+$ it holds

$$(x_{iC}, \omega_{iD}) \succsim_i (y_{iC}, \omega_{iD}) \iff (x_{iC}, \omega_{iD}) \succsim_i^* (y_{iC}, \omega_{iD})$$

$$(\omega_{iC}, x_{iD},) \succsim_i (\omega_{iC}, y_{iD}) \iff (\omega_{iC}, x_{iD},) \succsim_i^* (\omega_{iC}, y_{iD}),$$

and (p_C, p_D) strictly separates between $\sum_{i\in I}\omega_i$ and

$$\sum_{i\in I_1\cup I_3} R(\varphi_i(e|_C), \succsim_i (e^*)) + \sum_{i\in I_2} R(\varphi_i(e|_D), \succsim_i (e^*))$$

Since $\varphi(e^*|_C) = \varphi(e|_C)$ and $\varphi(e^*|_D) = \varphi(e|_D)$, No Loss from Trade requires

$$\begin{aligned}\sum_{i\in I}\varphi_i(e^*) &\in \sum_{i\in I_1\cup I_3} R(\varphi_i(e^*|_C), \succsim_i (e^*)) + \sum_{i\in I_2} R(\varphi_i(e^*|_D), \succsim_i (e^*))\\ &= \sum_{i\in I_1\cup I_3} R(\varphi_i(e|_C), \succsim_i (e^*)) + \sum_{i\in I_2} R(\varphi_i(e|_D), \succsim_i (e^*)),\end{aligned}$$

which is a contradiction to $\sum_{i \in I} \varphi_i(e^*) = \sum_{i \in I} \omega_i$. ■

Because it is generically impossible that Walrasian allocations are Pareto-ranked between subeconomies (provided that Walrasian correspondence is single-valued there), we obtain the result.

5 Conclusion

This paper initiates a formal study of trade liberalization in a market design context. We have asked a very basic question: whether, in fact, the invisible hand could be modified to guide agents in Pareto improvements when opening markets to new trade. We have shown that the answer is negative in a very broad sense.

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