Can everyone benefit from innovation?

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Abstract

We study a resource allocation problem with variable technologies, and ask if there is an allocation rule under which innovation hurts nobody. We consider the class of constant-returns-to-scale technologies. Together with efficiency and a natural participation condition that nobody should receive a worse allocation than what he can obtain alone, we run into a type of impossibility.

1 Introduction

Innovation is widely understood as beneficial as it enlarges the possibilities of what society can achieve. However, people are often concerned that they may lose out due to the rise in technology. For example, technology often renders certain types of labor obsolete, leading to unemployment of a class of society. Our aim in this note is to understand whether these concerns can be ameliorated by appropriately adjusting the market mechanism; or perhaps, by considering another type of mechanism altogether.

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In the following two examples we demonstrate formally the known fact that innovation can hurt certain people in the market. The key to this observation is that innovation changes relative prices. Such change may hurt an individual in the following way.

1. If she prefers consuming a good rather than using it as production input, innovation increases the factor demand for it and makes it more expensive. The associated negative welfare effect can be larger than the positive effect of making the output good cheaper.

2. If she relies on income from selling some input good, innovation which makes it dispensable decreases her income. The negative effect can be larger than the positive effect of making the output good cheaper.

We illustrate the first point with the following example.

Example 1 Suppose that there are two goods andn two individuals, i and j, who have identical preferences represented by

$$u(x_1, x_2) = x_1 x_2.$$

Individual i's initial endowment is $(1, 9)$ and j's is $(9, 1)$. When there is no production, competitive equilibrium yields

$$p_1 = 1, \quad x_i = (5, 5), \quad x_j = (5, 5)$$

where the price of Good 2 is normalized to 1.

Now suppose it becomes possible to produce Good 2 from Good 1 with constant returns, and the marginal productivity is 2. Then competitive equilibrium with arbitrary profit share (profit is zero in equilibrium anyway) yields

$$p_1 = 2, \quad x_i = \left(\frac{11}{4}, \frac{11}{2}\right), \quad x_j = \left(\frac{19}{4}, \frac{19}{2}\right).$$

$\frac{5}{2}$ units of Good 1 are used as input and 5 units of Good 2 are produced. Individual i loses.

The second point is illustrated by the following example.

Example 2 Suppose that there are three goods and two individuals, i and j. Individual i's initial endowment is $(9, 1, 1)$ and j's is $(1, 9, 1)$. They have identical preferences represented in the form

$$u(x_1, x_2, x_3) = x_1 x_2 x_3$$

There is a constant returns to scale technology in which Good 3 is produced from Good 1 and 2, which is described by

$$f(z_1, z_2) = z_1^{\frac{1}{2}} z_2^{\frac{1}{2}}.$$

Then competitive equilibrium with any sharing of technology yields

$$p_1 = p_2 = \frac{1}{2}, \quad x_i = (4, 4, 2), \quad x_j = (4, 4, 2)$$

Note that here 2 units of Good 3 are produced from 2 units of Good 1 and 2 units of Good 2.

Now consider the production technology given by

$$f^*(z_1, z_2) = \frac{\sqrt{2}}{4} z_1 + \frac{\sqrt{2}}{2} z_2$$

Note that this production possibility frontier nests the previous one.

Then competitive equilibrium yields

$$p_1 = \frac{\sqrt{2}}{4}, \quad p_2 = \frac{\sqrt{2}}{2}$$

and

$$x_i = \left(\frac{11 + 2\sqrt{2}}{3}, \frac{11 + 2\sqrt{2}}{6}, \frac{11\sqrt{2} + 4}{12} \right), \quad x_j = \left(\frac{19 + 2\sqrt{2}}{3}, \frac{19 + 2\sqrt{2}}{6}, \frac{19\sqrt{2} + 4}{12} \right)$$

i loses.

Suppose instead that we do not take the market mechanism as given, but ask whether there are other methods of allocating resources which avoid the pitfalls of the previous examples. We propose to study this question axiomatically. To this end, we study *social choice functions.* These objects map triples of preference profiles, endowment profiles and production technologies into feasible allocations.

We suppose that technologies exhibit *constant-returns-to-scale.* This restrictive hypothesis should, if anything, make positive results easier to obtain. McKenzie [4] argues that constant-returns-to-scale technologies are more fundamental than general convex technologies exhibiting diminishing returns, once we explicitly account for resources in production management.

We consider three axioms. First is Technology Monotonicity, the primary requirement that innovation should not hurt anybody. Second is Efficiency, requiring that any selected allocation must be Pareto-efficient. Third is Free Access Lower Bound, which requires that nobody should receive a worse allocation than what she could obtain by accessing the technology alone. Since in a production economy with constant returns the technology is replicable, the assumption that everybody can/should freely access the technology is reasonable. Hence the lower bound condition is understood as a natural participation constraint.

One can see that under Efficiency and Technology Monotonicity the solution induces a (weakly) monotone path solution, given preference profile and endowment profile: fix a utility representation profile, and fix a (weakly) monotone path in the utility space; for each production technology draw the corresponding utility possibility frontier and find the point at which the monotone path crosses the frontier. See, *e.g.* [15].

We show that any such (weakly) monotone path has to have unpleasant properties under Free Access Lower Bound, the natural participation condition. In particular, it has to have vertical or horizontal portions, along which somebody gains nothing from innovation (something like a dictatorship in a region of utility space). There are two kinds of difficulties here. One is that an individual, who prefers to consume input goods rather than using them for production, may gain nothing from innovation until it exceeds some threshold at which he starts to prefer production. Until then, he gains nothing even from exchange activity. Second is that one may gain nothing from innovation until it exceeds a threshold at which there is no need to coordinate how to combine production inputs. Until then, he gains nothing either from exchange activity or from coordination of inputs.

Everybody may start gaining from innovation when it reaches the threshold as explained. However, the welfare gain for the disadvantaged individuals can be made only in a slow-starting manner.

Related Literature

Technology Monotonicity was introduced by Roemer [12] and he imposed this as one of the axioms in his characterization of welfare egalitarianism.

Moulin [9, 5] considered a production economy in which there is one input good and one output good, where technology exhibits either decreasing returns to scale or increasing returns to scale. In each setting, he characterized the solution, constant-returns-to-scale-equivalence solution, which satisfies Technology Monotonicity, Efficiency and an axiom stating that nobody should receive a better (resp. worse) allocation than he can get by freely accessing the technology where endowments are equally divided. Note that the lower bound condition there is essentially equivalent to our Free Access Lower Bound.[1] Since we obtain impossibilities with the three axioms of Technology Monotonicity, Efficiency and Free Access Lower Bound, it is seen that whether a production economy involves exchange activity and/or

[1] See also Moulin [6, 7] for results in the setting of public good provision.

coordination of how to combine multiple inputs makes a critical difference.

There are axiomatic studies of solidarity conditions with respect to other kinds of economic changes. They show that it is hard to reconcile the idea of solidarity with efficiency of allocations when we also require a natural condition on distributive justice, participation or operationality, such as welfare lower bound, informational efficiency or path independence.

Moulin and Thomson [11] considered an exchange economy starting with social endowments, and asked if everyone can benefit from an increase in the social endowment vector. The property was called Resource Monotonicity. They showed that there is no allocation rule which satisfies Resource Monotonicity, Efficiency and Equal-Division Lower Bound: the requirement that nobody should receive worse allocation than equal division. They further established an impossibility when Equal-Division Lower Bound is replaced by the requirement that nobody's allocation should be dominated by anybody else's.

Their impossibility result of requiring three axioms of monotonicity, efficiency and welfare lower bound is reminiscent of our result.

Chambers and Hayashi [1] considered exchange economies, in which the set of tradable goods are variable, and asked if everyone can benefit from opening markets for goods which had not been tradable. This requirement was termed No Loss from Trade. They showed that No Loss from Trade, Efficiency for any given set of tradable goods and Independence of Untraded Commodities, an informational efficiency requirement that allocation should depend only on preferences induced over tradable goods, imply that only one person can gain from trade at earlier steps of trade liberalization.

Chambers and Hayashi [2] considered exchange economies with variable populations, and asked if everyone can benefit from integrating economies. This requirement was termed Integration Monotonicity. An idea of path independence is built in the setting, as a larger economy might have come from many different ways of economic integration. They showed that Integration Monotonicity and Efficiency imply that allocation must be in the core in any economy. Because of the core convergence theorem, any such allocation must converge to a competitive allocation after replications. And the competitive solution easily fails to satisfy Integration Monotonicity.

In the class of transferable utility games, Integration Monotonicity is equivalent to Population Monotonicity introduced by Sprumont [13]. He characterized the class of TU games which admits population-monotonic payoff configurations, and showed that impossibility is obtained with a smaller number of individuals when core is small.

Population Monotonicity, a different axiom, was introduced by Thomson [16], who considered allocating a fixed amount of resources among variable numbers of individuals. It is a solidarity requirement that everybody should lose together when there are new participants. See Sprumont [14] for a detailed survey.

In the setting of allocating private goods with fixed social endowments, Thomson [17] showed that there is a population-monotonic and efficient allocation rule, while Moulin [8] suggested that we reach impossibility if we additionally impose envy-freeness. Kim [3] gave a formal proof. In the setting of allocating fixed amounts of private goods and a fixed amount of numeraire good, where preferences are linear in the numeraire good, Moulin [10] showed that in general there is no population monotonic and efficient allocation rule. He showed that when preferences exhibit substitutability, the Shapley value is population-monotonic.

2 Setting and axioms

Let $I = \{1, \cdots, n\}$ be the set of individuals. The number of goods is denoted by l.

Let $\mathcal{R}$ be the set of preference orderings over $\mathbb{R}^l_+$, which are complete, transitive, continuous, convex and strongly monotone. Mostly, we are interested in differentiable preferences satisfying a simple boundary condition. This condition works as follows: given R_i for individual i which a utility representation u_i differentiable on $\mathbb{R}^l_{++}$, let $MRS_i^{k,h}(x_i)$ denote the marginal rate of substitution of Good h for Good k for i at $x_i \in \mathbb{R}^l_{++}$, which is given by

$$MRS_i^{k,h}(x_i) = \frac{\frac{\partial u_i(x_i)}{\partial x_{ik}}}{\frac{\partial u_i(x_i)}{\partial x_{ih}}}.$$

Then the boundary condition is:

$$\lim_{x_{ik}\to 0} MRS_i^{k,h}(x_i) = \infty, \quad \lim_{x_{ih}\to 0} MRS_i^{k,h}(x_i) = 0$$

for all $k \neq h$.

Let $\mathcal{Y}$ be the set of *constant-returns-to-scale* technologies. That is, $Y \in \mathcal{Y}$ holds if and only if

1. $Y \subset \mathbb{R}^l$;
2. $Y \cap \mathbb{R}^l_+ = \{\mathbf{0}\}$;
3. $Y \supset -\mathbb{R}^l_+$;
4. Y is closed and convex;
5. for all $y \in Y$ and $\lambda \geq 0$ it holds $\lambda y \in Y$.

Each individual's initial endowment is denoted by $\omega_i \in \mathbb{R}^l_{++}$ for each $i \in I$, and a list of endowments is denoted by $\omega = (\omega_1, \cdots, \omega_n) \in \mathbb{R}^{nl}_{++}$.[2]

An *economy* is a triple $(R, \omega, Y) \in \mathcal{R}^I \times \mathbb{R}^{nl}_{++} \times \mathcal{Y}$, which consists of preference profile $R = (R_1, \cdots, R_n)$ and endowment profile $\omega = (\omega_1, \cdots, \omega_n)$ and production set Y.

[2] One may allow nonpositive endowments, as far as some input goods are productive, but we assume strictly positive endowments for simplicity.

A *social choice function* $\varphi : \mathcal{R}^I \times \mathbb{R}^{nl}_{++} \times \mathcal{Y} \to \mathbb{R}^{nl}_{+}$ is a mapping such that

$$\sum_{i \in I} \varphi_i(R, \omega, Y) - \sum_{i \in I} \omega_i \in Y$$

for all $(R, \omega, Y) \in \mathcal{R}^I \times \mathbb{R}^{nl}_{++} \times \mathcal{Y}$.

We impose the following three axioms.

Axiom 1 (**Technology Monotonicity**): For all $R \in \mathcal{R}^I$, $\omega \in \mathbb{R}^{nl}_{++}$ and $Y, Y' \in \mathcal{Y}$ with $Y \subset Y'$, it holds

$$\varphi_i(R, \omega, Y') \ R_i \ \varphi_i(R, \omega, Y)$$

for all $i \in I$.

Axiom 2 (**Efficiency**): For all $R \in \mathcal{R}^I$, $\omega \in \mathbb{R}^{nl}_{++}$ and $Y \in \mathcal{Y}$, $\varphi(R, \omega, Y)$ is Pareto-efficient.

Axiom 3 (**Free Access Lower Bound**): For all $\omega \in \mathbb{R}^{nl}_{++}$, there is $\underline{\omega} \in \mathbb{R}^{nl}_{++}$ with $\sum_{i \in I} \underline{\omega}_i = \sum_{i \in I} \omega_i$ such that for all $R \in \mathcal{R}^I$ and $Y \in \mathcal{Y}$ it holds

$$\varphi_i(R, \omega, Y) \ R_i \ \underline{\omega}_i + y_i$$

for all $y_i \in Y$ with $\underline{\omega}_i + y_i \in \mathbb{R}^l_+$, for all $i \in I$.

This axiom generalizes two special ones, one in which $\underline{\omega} = \left(\frac{1}{n} \sum_{i \in I} \omega_i, \cdots, \frac{1}{n} \sum_{i \in I} \omega_i\right)$ and the other in which $\underline{\omega} = \omega$. The first one corresponds to the requirement that only the social endowment should matter and each individual must receive equal-division as his endowment. The second corresponds to the requirement that we should take the initial endowment profile as given.

There are two motivations for Free Access Lower Bound. One is normative. Because technology in this setting is replicable, it is desirable that everybody is allowed to access to it. Second is descriptive, since Free Access

Lower Bound is interpreted as a participation constraint on exchange activity and production coordination, given that technology is replicable and nobody can be excluded from accessing it.

Here is a prominent example of solution satisfying Free Access Lower Bound, in which each individual simply maximizes her utility using the technology and her endowment.

Example 3 The *free access solution* FA is defined as follows. Given any $\omega \in \mathbb{R}^{nl}_{++}$, fix some $\underline{\omega} \in \mathbb{R}^{nl}_{++}$ with $\sum_{i \in I} \underline{\omega}_i = \sum_{i \in I} \omega_i$. Then for every $R \in \mathcal{R}^I$ and $Y \in \mathcal{Y}$ let $FA_i(R, \omega, Y) = \underline{\omega}_i + y_i$ with $y_i \in Y$, $\underline{\omega}_i + y_i \in \mathbb{R}^l_+$, for each $i \in I$, which satisfies

$$\underline{\omega}_i + y_i \ R_i \ \underline{\omega}_i + y_i'$$

for all $y_i' \in Y$ with $\underline{\omega}_i + y_i' \in \mathbb{R}^l_+$.

Let $y_i^{FA}(R, \omega, Y)$ denote the production activity for i associated with $FA_i(R, \omega, Y)$, and let $z_i^{FA}(R, \omega, Y)$ denote the input vector for i when the set of input goods is prespecified.

This satisfies Technology Monotonicity, Free Access Lower Bound but fails Efficiency. However, Efficiency is met when the technology is linear and everybody chooses strictly positive production activity and strictly positive consumption vectors.

More generally, the solution is inefficient, since it

1. lacks exchange activity; and

2. lacks social coordination of production activity.

Here we offer solution which satisfies Technology Monotonicity and Efficiency.

Example 4 *Monotone path solutions* are defined as follows. Given a preference profile $R \in \mathcal{R}^I$, fix a profile of continuous utility representations

$u[R] = (u_1[R_1] \cdots, u_n[R_n])$. Fix $\omega \in \mathbb{R}^{nl}_{++}$, which may potentially affect how we draw a path.

Then fix a weakly monotone and continuous path $\Phi(u[R], \omega)$ in $\mathbb{R}^n$.

For every $Y \in \mathcal{Y}$ define the utility possibility set

$$U(Y; u[R], \omega) = \left\{ u[R](x) = (u_1[R_1](x_1), \cdots, u_n[R_n](x_n)) \in \mathbb{R}^n : \sum_{i \in I} x_i - \sum_{i \in I} \omega_i \in Y \right\}$$

Then $\Phi(u[R], \omega) \cap U(Y; u[R], \omega)$ has a unique largest vector element, which is denoted by $\max \Phi(u[R], \omega) \cap U(Y; u[R], \omega)$. Then one can define φ by taking $\varphi(R, \omega, Y)$ as an allocation satisfying

$$u[R](\varphi(R, \omega, Y)) = \max \Phi(u[R], \omega) \cap U(Y; u[R], \omega)$$

where such an allocation is unique up to Pareto indifference, and physically unique when preferences are strictly convex.

The monotone path solution satisfies Efficiency and Technology Monotonicity.

Conversely, any social choice function φ satisfying Efficiency and Technology Monotonicity induces a (weakly) monotone path solution, where the monotone path $\Phi(u[R], \omega)$ given u, a fixed utility representation for R, and endowment profile ω is defined by

$$\Phi(u[R], \omega) = \{u[R](\varphi(R, \omega, Y)) \in \mathbb{R}^n : Y \in \mathcal{Y}\}.$$

In the next section, however, we show that under Free Access Lower Bound, a natural participation condition, the monotone path so described necessarily has unpleasant properties.

3 Two difficulties

3.1 One may gain nothing until he starts to prefer production

Proposition 1 Let $I = \{i, j\}$ and $l = 2$. Restrict attention to the class of constant returns to scale technologies in which one good (say Good 2) is produced from the other (Good 1), where marginal productivity is denoted by $a \geq 0$ and Y_a denotes the technology given by a. Pick any $\omega \in \mathbb{R}^{2\times 2}_{++}$, let $\underline{\omega} \in \mathbb{R}^{2\times 2}_{++}$ denote the endowment as described in Free Access Lower Bound.

Pick any differentiable preference profile $R \in \mathcal{R}^2$ satisfying the standard boundary condition. Without loss of generality, assume $MRS^{1,2}_i(\underline{\omega}_i) \geq MRS^{1,2}_j(\underline{\omega}_j)$.

Then for all $a \in [0, MRS^{1,2}_i(\underline{\omega}_i)]$ it holds

$$\varphi_i(R, \omega, Y_a)\ I_i\ \underline{\omega}_i.$$

and j obtains their optimal bundle subject to this constraint.

Proof. By assumption, we have $MRS^{1,2}_i(\underline{\omega}_i) \geq MRS^{1,2}_j(\underline{\omega}_j)$.

Now consider

$$a^* = MRS^{1,2}_i(\underline{\omega}_i).$$

Then, in economy (R, ω, Y_{a^*}) the only allocation meeting Free Access Lower Bound is $FA(R, \omega, Y_{a^*})$, where $FA_i(R, \omega, Y_{a^*}) = \underline{\omega}_i$ and $FA_j(R, \omega, Y_{a^*}) \gg \mathbf{0}$. Note that the latter follows from the boundary condition.

From Technology Monotonicity, it must hold

$$\underline{\omega}_i = \varphi_i(R, \omega, Y_{a^*})\ R_i\ \varphi_i(R, \omega, Y_0)$$

but in economy (R, ω, Y_0) Free Access Lower Bound implies

$$\varphi_i(R, \omega, Y_0)\ R_i\ \underline{\omega}_i.$$

Thus we obtain

$$\varphi_i(R, \omega, Y_0)\ I_i\ \underline{\omega}_i.$$

By Efficiency $\varphi(R, \omega, Y_0)$ is the point on i's indifference curve passing through $\underline{\omega}_i$ which is optimal for j.

By Technology Monotonicity, this property extends to all $a \in [0, MRS_i^{1,2}(\underline{\omega}_i)]$.

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Geometrically speaking, the path connecting utility pairs in the utility space must be vertical as the utility possibility frontier expands when a goes from 0 to $MRS_i^{1,2}(\underline{\omega}_i)$. Hence if $MRS_i^{1,2}(\underline{\omega}_i)$ is large, the vertical potion must be long. This means that i can hardly enjoy any welfare gain from innovation.

In other words, in order that i should not lose because of innovation the possibility of his gain from exchange must be excluded in the outset.

Now if we can also think of a class of opposite technologies in which Good 1 is produced from Good 2, we can apply the same argument in the opposite manner. Let $b \geq 0$ denote marginal productivity and Y_b denotes the technology given by b.

Then for all $b \in [0, 1/MRS_j^{1,2}(\underline{\omega}_j)]$ it holds

$$\varphi_j(R, \omega, Y_b)\ I_j\ \underline{\omega}_j.$$

and i obtains optimal allocation under this constraint.

Then by combining the two argument we obtain

$$\varphi_i(R, \omega, Y_0)\ I_i\ \underline{\omega}_i, \quad \varphi_j(R, \omega, Y_0)\ I_j\ \underline{\omega}_j$$

which typically contradicts Efficiency.

Thus we obtain

Proposition 2 Assume that there are two goods and two individuals. Assume that $\mathcal{Y}$ includes two classes of constant returns to scale technologies, one in which Good 1 is produced from Good 2, the other in which Good 1 is produced from Good 2.

Then there is no allocation rule which satisfies Efficiency, Technology Monotonicity and Free Access Lower Bound.

3.2 One may gain nothing until there is no need for coordination on combining inputs

Now suppose that there are three goods.

Proposition 3 Let $I = \{i, j\}$ and $l = 3$. Restrict attention to the class of constant returns to scale technologies in which one (say Good 3) is produced from the others (Good 1 and 2). Given any $\omega \in \mathbb{R}^{2\times 3}_{++}$, let $\underline{\omega} \in \mathbb{R}^{2\times 3}_{++}$ denote the endowment as described in Free Access Lower Bound.

Fix any technology Y given by a production function $f : \mathbb{R}^2_+ \to \mathbb{R}_+$ which is differentiable on $\mathbb{R}^2_{++}$.

Pick any $R \in \mathcal{R}^2$ such that $z_i^{FA}(R, \omega, Y), z_j^{FA}(R, \omega, Y) \gg \mathbf{0}$. Without loss of generality, assume $MRS_i^{1,2}(FA_i(R, \omega, Y)) \geq MRS_j^{1,2}(FA_j(R, \omega, Y))$.

Let Y^* be the linear technology given by production function

$$f^*(z_1, z_2) = f_1(z_i^{FA}(R, \omega, Y))z_1 + f_2(z_i^{FA}(R, \omega, Y))z_2.$$

Suppose that $z_j^{FA}(R, \omega, Y^*) \gg \mathbf{0}$.

Then for all $Y \subset Y' \subset Y^*$ it holds

$$\varphi_i(R, \omega, Y') \; I_i \; FA_i(R, \omega, Y).$$

and j obtains their optimal allocation subject to this constraint.

Proof. Observe that $FA_i(R, \omega, Y), FA_j(R, \omega, Y) \gg \mathbf{0}$ follows from the boundary condition.

In economy (R, ω, Y^*) the only allocation meeting Free Access Lower Bound is $FA(R, \omega, Y^*)$, and note that $FA_i(R, \omega, Y^*) = FA_i(R, \omega, Y)$. Note that $FA_i(R, \omega, Y^*), FA_j(R, \omega, Y^*) \gg \mathbf{0}$ follows from the boundary condition.

From Technology Monotonicity, we have

$$FA_i(R, \omega, Y) = \varphi_i(R, \omega, Y^*) R_i \varphi_i(R, \omega, Y)$$

but in economy (R, ω, Y) Free Access Lower Bound implies

$$\varphi_i(R, \omega, Y) \; R_i \; FA_i(R, \omega, Y)$$

Thus we obtain

$$\varphi_i(R, \omega, Y) \; I_i \; FA_i(R, \omega, Y)$$

By Efficiency $\varphi(R, \omega, Y)$ is the point on i's indiference curve passing through $FA_i(R, \omega, Y)$ which is optimal for j.

By Technology Monotonicity, this property extends to all $Y \subset Y' \subset Y^*$.

■

Now let Y^{**} be the linear technology given by production function

$$f^{**}(z_1, z_2) = f_1(z_j^{FA}(R, \omega, Y))z_1 + f_2(z_j^{FA}(R, \omega, Y))z_2$$

Then by construction it holds $FA_j(R, \omega, Y^{**}) = FA_j(R, \omega, Y)$.

One can choose the (R, ω, Y) suitably so that $z_i^{FA}(R, \omega, Y^{**}) \gg 0$.

Thus by applying the same argument for all $Y \subset Y' \subset Y^{**}$, we have

$$\varphi_j(R, \omega, Y') I_j FA_j(R, \omega, Y),$$

and i obtains their optimal allocation subject to this constraint.

Then by combining the two arguments, we obtain

$$\varphi_i(R, \omega, Y) I_i FA_i(R, \omega, Y), \quad \varphi_j(R, \omega, Y) I_j FA_j(R, \omega, Y)$$

which typically contradicts to Efficiency.

Thus we obtain

Proposition 4 Assume that there are three goods and two individuals. Assume that $\mathcal{Y}$ includes the class of constant returns to scale technologies, in which one fixed good (say Good 3) is produced from the other two (Good 1 and 2).

Then there is no allocation rule which satisfies Efficiency, Technology Monotonicity and Free Access Lower Bound.

4 Conclusion

We have studied a resource allocation problem with variable technologies and asked if there is an allocation rule under which innovation hurts nobody. The requirement is presented as an axiom called Technology Monotonicity. We considered the class of constant-returns-to-scale technologies, as it is fundamental when all the relevant factors are taken into account.

First we showed that the competitive solution fails Technology Monotonicity, because of two reasons. One is that innovation makes an input good more expensive and it hurts an individual who wants to consume it rather than using it as an input in production. The other is that innovation makes some input goods dispensable and hurts an individual who relies on income from selling it.

Then we considered a social choice problem, without taking the market solution as given, and considered two additional axioms, Efficiency and Free Access Lower Bound. The latter is a natural participation condition, requiring that nobody should receive a worse allocation than what he could obtain by accessing the technology by himself alone.

We have shown that Technology Monotonicity, Efficiency and Equal Acess Lower Bound imply that that somebody can gain nothing until innovation exceeds certain threshold.

We pointed out two difficulties. One is that when somebody prefers to consume input goods rather than using them for production, he may gain nothing from innovation until it exceeds a threshold at which he starts to prefer production. Second is that somebody may gain nothing from innovation until it exceeds a threshold at which there is no need to coordinate how to combine production inputs.

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