Preference Aggregation under Uncertainty: Savage vs. Pareto

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Abstract

Following Mongin [13, 14], we study social aggregation of subjective expected utility preferences in a Savage framework. We argue that each of Savage's P3 and P4 are incompatible with the strong Pareto property. A representation theorem for social preferences satisfying Pareto indifference and conforming to the state-dependent expected utility model is provided.

1 Introduction

Harsanyi's theorem [9] discusses social aggregation of individual preferences in a risky environment. He shows that when all agents' preferences conform to the axioms of expected utility, if social preference also conforms to the axioms of expected utility and satisfies Pareto indifference with respect to agents' preferences, then social preference can be represented as a weighted

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sum of agents' expected utility functions.[1] An example due to Diamond [6] illustrates that Harsanyi's assumptions on social preference may not be compelling.

Mongin [13, 14] pursues a similar program within Savage's [17] framework of subjective uncertainty. If all agents behave according to Savage's axioms, he shows that it is generally impossible for social preference to jointly satisfy Savage's axioms and the strong Pareto property with respect to individual preferences. Two natural responses to this negative conclusion present themselves. One is to condemn the strong Pareto property in a setting of subjective uncertainty. Gilboa, Samet, and Schmeidler [8] follow this approach, see also Raiffa ([16] p. 228-238) and Broome [4]. Our approach is to analyze the general structure underlying the impossibility results, by studying them at an axiomatic level.

At an individual level, Savage's axioms are used to deliver a personal notion of probability. The primary axioms in his theory are P2 (*sure-thing principle*), P3 (*eventwise monotonicity*), and P4 (*weak comparative probability*).[2] In terms of empirical violations of Savage's theory at an individual level, P2 is often criticized. We have no specific criticism against assuming P2 at a social level. Instead, we discuss P3 and P4. We remain agnostic on the normative appeal of these axioms, as we believe that there are reasons both *for* and *against* each of them. However, we show by examples that *each* of these axioms conflicts with the strong Pareto property on its own.

Our work concludes by studying the most well-known model which generically violates P3 and P4, the state-dependent expected utility model. A similar exercise is conducted by Mongin [14], in the Anscombe-Aumann [1] and Karni, Schmeidler, and Vind [11] frameworks.

We provide a representation theorem for social preference conforming to the state-dependent model and which satisfies Pareto indifference with respect to individual preferences. Under Pareto indifference, social preference can be represented by a weighted sum of individual utility representations.

Section 2 describes the model and provides examples and propositions illustrating the conflict between Savage's axioms and the Pareto properties. Section 3 provides a representation theorem in a state-dependent expected utility framework. Section 4 discusses the relation of our work with that of

[1] For more on Harsanyi's theorem, see Mongin and D'Aspremont [15] and Weymark [19].

[2] The terminology is from Machina and Schmeidler [12].

Gilboa, Samet, and Schmeidler. Finally, Section 5 concludes.

2 The model and Savage's axioms

Let (S, Σ) be a measurable space, where S is a set of **states** and Σ is a σ-algebra of **events**. We use s to denote a generic state and A or B to denote generic events. Let X be a set of **outcomes**. Define the set of (simple) **acts** $\mathcal{F}$ as the set of finite-ranged Σ-measurable mappings $f : S \to X$. We use f or g to denote a generic act. For $x, y \in X$, $A \in \Sigma$, define $xAy \in \mathcal{F}$ as

$$xAy \equiv \begin{pmatrix} x \text{ if } s \in A \\ y \text{ if } s \in A^c \end{pmatrix}.$$

With a slight abuse of notation, $x \in X$ also denotes the act whose constant outcome is x.

Savage's axioms apply to binary relations $\succeq$ on $\mathcal{F}$. Say that $A \in \Sigma$ is **null** for $\succeq$ if for all $x \in X$ and all $f \in \mathcal{F}$,

$$f \sim \begin{pmatrix} x \text{ if } s \in A \\ f(s) \text{ otherwise} \end{pmatrix}.$$

For a set of agents $N \equiv \{1, ..., n\}$, agent i's preference is denoted by $\succeq_i$ and social preference is denoted by $\succeq_0$. Say that $\succeq_0$ **satisfies Pareto-indifference with respect to** $(\succeq_1, ..., \succeq_n)$ if for all $f, g \in \mathcal{F}$, if for all $i \in N$, $f \sim_i g$, then $f \sim_0 g$. Say that $\succeq_0$ **satisfies the weak Pareto property with respect to** $(\succeq_1, ..., \succeq_n)$ if for all $f, g \in \mathcal{F}$, if for all $i \in N$, $f \succ_i g$, then $f \succ_0 g$. Say that $\succeq_0$ **satisfies the strong Pareto property with respect to** $(\succeq_1, ..., \succeq_n)$ if it satisfies Pareto indifference with respect to $(\succeq_1, ..., \succeq_n)$ and if for all $i \in N$, $f \succeq_i g$, with strict preference for some $j \in N$, then $f \succ_0 g$.

A function $U : \mathcal{F} \to \mathbb{R}$ is a **subjective expected utility functional** if there exists a nonatomic, countably additive[3] probability measure μ on (S, Σ) and a function $u : X \to \mathbb{R}$ such that for all $f \in \mathcal{F}$, $U(f) = \int_S u(f(s))\, d\mu(s)$. Say a binary relation $\succeq$ is a **subjective expected utility preference** if there exists a subjective expected utility functional U such that for all $f, g \in \mathcal{F}$,

$$f \succeq g \Leftrightarrow U(f) \geq U(g).$$

[3] Savage's theorem [17] only delivers a finitely additive probability measure. Arrow [2] discusses conditions which guarantee that the probability measure is countably additive.

For any such U representing a nondegenerate subjective expected utility preference, the associated probability measure μ is unique, and the function u is unique up to positive affine transformation.

2.1 P3: Eventwise monotonicity

Savage's axiom P3 states:

P3 For all non-null $A \in \Sigma$, $x, y \in X$, $f \in \mathcal{F}$,

$$x \succeq y \Leftrightarrow \begin{pmatrix} x \text{ if } s \in A \\ f(s) \text{ if } s \in A^c \end{pmatrix} \succeq \begin{pmatrix} y \text{ if } s \in A \\ f(s) \text{ if } s \in A^c \end{pmatrix}.$$

A social preference that satisfies P3 cannot generally satisfy the weak Pareto property. The following example illustrates this point.

Example 1 Let $N \equiv \{1, 2\}$ and $X \equiv \mathbb{R}^2_+$. The set X represents distributions of wealth amongst the two agents. Let $\succeq_0$ be a social preference over $\mathcal{F}$. Assuming each agent likes more wealth to less and cares only about her own wealth, a plausible social ranking is $(0, 100) \sim_0 (100, 0)$. Let $A, A^c \in \Sigma$. Suppose agent one believes A is more likely than A^c and agent two believes A^c is more likely than A. Then for $i = 1, 2$,

$$(100, 0)\, A\, (0, 100) \succ_i (0, 100)\, A\, (100, 0).$$

By the weak Pareto property,

$$(100, 0)\, A\, (0, 100) \succ_0 (0, 100)\, A\, (100, 0).$$

Here, $\succeq_0$ violates P3. To see this, suppose $\succeq_0$ satisfies P3. As $(100, 0) \sim_0 (0, 100)$, and as $\succeq_0$ satisfies P3,

$$(100, 0) \sim_0 (100, 0)\, A\, (0, 100)$$

and

$$(0, 100)\, A\, (100, 0) \sim_0 (0, 100).$$

By transitivity, the ranking $(100, 0) \succ_0 (0, 100)$ holds, a contradiction.

2.2 P4: Weak comparative probability

Savage's axiom P4 states:

P4 For all $\overline{x}, \underline{x}, \overline{y}, \underline{y}$ such that $\overline{x} \succ \underline{x}$ and $\overline{y} \succ \underline{y}$, for all $A, B \in \Sigma$

$$\overline{x}A\underline{x} \succ \overline{x}B\underline{x} \Leftrightarrow \overline{y}A\underline{y} \succ \overline{y}B\underline{y}.$$

Example 2 shows that P4 is also incompatible with the strong Pareto property.

Example 2 Let $N \equiv \{1,2\}$ and $X \equiv R_+^2$, where the numerical quantities again represent monetary values. Each agent only cares about the amount she receives. Suppose again that agent one believes A is more likely than A^c, and agent two believes A^c is more likely than A. By strong Pareto, $(100,0) \succ_0 (0,0)$ and $(0,100) \succ_0 (0,0)$. Thus,

$$(100,0)\,A\,(0,0) \succ_1 (0,0)\,A\,(100,0)$$

and

$$(100,0)\,A\,(0,0) \sim_2 (0,0)\,A\,(100,0)\,.$$

By the strong Pareto property,

$$(100,0)\,A\,(0,0) \succ_0 (0,0)\,A\,(100,0)\,.$$

By a symmetric argument,

$$(0,100)\,A\,(0,0) \prec_0 (0,0)\,A\,(0,100)\,.$$

These rankings clearly violate P4.

2.3 General incompatibility results

The following propositions are general versions of the examples.

The first proposition follows directly from Example 1. For any preference relation $\succeq$ satisfying P4, a **likelihood relation** $\succeq^l$ on Σ can be defined as follows: $A \succeq^l B$ if and only if there exist $x, x' \in X$ such that $x \succ x'$ and $xAx' \succeq xBx'$.[4]

[4] Propositions 1, 2, and 3 are stated for environments with only two agents. Similar results can be established for any number of agents, by partitioning the set of agents into two types, where each type has a preference relation corresponding to one of the two preference relations in the propositions.

Proposition 1: Suppose $\succeq_1$ and $\succeq_2$ satisfy P4 and for all $i = 1, 2$, $A \succeq_i^l B \Leftrightarrow B^c \succeq_i^l A^c$. Suppose there exist $\overline{x}, \underline{x} \in X$ such that $\overline{x} \succ_1 \underline{x}$, $\underline{x} \succ_2 \overline{x}$, and $\overline{x} \sim_0 \underline{x}$. If $\succeq_0$ satisfies P3 and the strong Pareto property with respect to $(\succeq_1, \succeq_2)$, then $\succeq_1^l = \succeq_2^l$.

The next proposition illustrates a related point. If individuals' "beliefs" are different, then aggregation under the strong Pareto property and P3 is possible only if their "tastes" are the same.

Proposition 2: Suppose $\succeq_1$ and $\succeq_2$ satisfy P3 and that there exist $\overline{x}, \underline{x}, \overline{y}, \underline{y}$ such that $\overline{x} \succ_1 \underline{x}$ and $\overline{y} \succ_2 \underline{y}$. Suppose there exist $A \in \Sigma$ such that A is non-null for $\succeq_1$ and null for $\succeq_2$, and $B \in \Sigma$ such that B is non-null for $\succeq_2$ and null for $\succeq_1$. If $\succeq_0$ satisfies P3 and satisfies the strong Pareto property with respect to $(\succeq_1, \succeq_2)$, then $\succeq_1 |_X = \succeq_2 |_X$.

Proof: We first show that each of A and B is non-null for $\succeq_0$. As $\succeq_1$ satisfies P3 and A is non-null for $\succeq_1$, $\overline{x}A\underline{x} \succ_1 \underline{x}$. As A is null for $\succeq_2$, $\overline{x}A\underline{x} \sim_2 \underline{x}$. By the strong Pareto property, $\overline{x}A\underline{x} \succ_0 \underline{x}$, so that A is non-null for $\succeq_0$. The proof for B is symmetric.

Suppose the statement of the proposition is false. Thus, $\succeq_1 |_X \neq \succeq_2 |_X$, and without loss of generality, there exist $x, y \in X$ such that $x \succeq_1 y$ and $y \succ_2 x$. As $\succeq_1$ satisfies P3, $xAy \succeq_1 y$, and as A is null for $\succeq_2$, $xAy \sim_2 y$. By the strong Pareto property, $xAy \succeq_0 y$. As A is non-null for $\succeq_0$, by P3, $x \succeq_0 y$.

As $\succeq_2$ satisfies P3 and as B is non-null for $\succeq_2$, $y \succ_2 xBy$, and as B is null for agent $\succeq_1$, $y \sim_1 xBy$. By the strong Pareto property, $y \succ_0 xBy$. As B is non-null for $\succeq_0$, by P3, $y \succ_0 x$. But we previously concluded that $x \succeq_0 y$, a contradiction.■

The last proposition follows directly from Example 2.

Proposition 3: Suppose $\succeq_1$ and $\succeq_2$ satisfy P4 and for all $i = 1, 2$, for all $x, x' \in X$, $A, B \in \Sigma$,

$$x \sim_i x' \Longrightarrow xAx' \sim_i xBx'.$$

Suppose there exist $\overline{x}, \underline{x}, \overline{y}, \underline{y} \in X$ such that $\overline{x} \succ_1 \underline{x}$,$\overline{x} \sim_2 \underline{x}$, $\overline{y} \sim_1 \underline{y}$, $\overline{y} \succ_2 \underline{y}$. If $\succeq_0$ satisfies P4 and the strong Pareto property with respect to $(\succeq_1, \succeq_2)$, then $\succeq_1^l = \succeq_2^l$.

3 A possibility result

We show that if social preference is not required to satisfy P3 and P4, Paretian aggregation is possible under the state-dependent expected utility model. To this end, say $U : \mathcal{F} \to \mathbb{R}$ is a **state-dependent subjective expected utility functional** if there exists a nonatomic, countably additive probability measure μ on (S, Σ) and a function $u : X \times S \to \mathbb{R}$ such that for all $x \in X$, $u(x, \cdot) : S \to \mathbb{R}$ is Σ-measurable and μ-integrable and for all $f \in \mathcal{F}$, $U(f) = \int_S u(f(s), s)\, d\mu(s)$. Say a binary relation $\succeq$ is a **state-dependent subjective expected utility preference** if there exists a state-dependent subjective expected utility functional U such that for all $f, g \in \mathcal{F}$,

$$f \succeq g \Leftrightarrow U(f) \geq U(g).$$

For more information about state-dependent subjective expected utility preferences, see Wakker and Zank [18].

The probability measure component of a state-dependent subjective expected utility functional is not unique. Thus, we cannot refer to society's "beliefs" in an unambiguous way. This is an artifact of the incompatibility of P4 with Paretian aggregation.

Theorem 1: Suppose that for all $i \in N$, $\succeq_i$ is a subjective expected utility preference represented by $U_i : \mathcal{F} \to \mathbb{R}$. Then $\succeq_0$ is a state-dependent subjective expected utility preference which satisfies Pareto-indifference with respect to $(\succeq_1, ..., \succeq_n)$ if and only if there exists a vector $\lambda \in \mathbb{R}^N$ such that for all $f, g \in \mathcal{F}$,

$$f \succeq_0 g \Leftrightarrow \sum_N \lambda_i U_i(f) \geq \sum_N \lambda_i U_i(g).$$

Representations corresponding to stronger Pareto properties can similarly be derived using Lemma 1 of Appendix A and the general representation theorems of DeMeyer and Mongin [5]. Mongin [14] proves a related theorem using the added structure of the Anscombe-Aumann [1] framework. However, an example (using probability measures with atoms) due to Mongin [13] illustrates that such an aggregation theorem is not generally true in a Savage framework.

The proof of Theorem 1 is in the Appendix. In order to prove Theorem 1, we establish in Lemma 1 that the utility possibilities set for a collection of state-dependent expected utility maximizers is convex, a fact which comes for free in the Anscombe-Aumann model. After establishing this, we apply a general representation theorem of DeMeyer and Mongin [5].

4 Gilboa, Samet, and Schmeidler

A work closely related to ours is that of Gilboa, Samet, and Schmeidler [8]. These authors take the position that the Pareto properties we study are not compelling at a social level. By applying Pareto indifference only when all agents agree on the probabilities of "relevant" events, they obtain an aggregation possibility result and representation. Their contribution is to show that by suitably weakening Pareto indifference, social preference can be made to generally satisfy Savage's axioms.

We do not take the position that Pareto indifference is universally compelling; nor do we take the position that P3 and P4 are generally not compelling. Our main contribution is to investigate the compatibility of various axioms at a basic level. Paralleling Gilboa, Samet, and Schmeidler, we weaken, as little as possible, the hypothesis that social preference should satisfy Savage's axioms, while maintaining Pareto indifference.

5 Conclusion

Mongin's [13] negative results make clear that Paretian aggregation is incompatible with the expected utility model for social preference. A natural conjecture is that Savage's P2 is the source of this impossibility result. Thus, Blackorby, Donaldson, and Mongin [3] study Paretian aggregation in non-expected utility models (in a multi-profile framework–see also Hylland and Zeckhauser [10]), establishing negative results. Our first contribution is to work at a primitive level, identifying which of Savage's axioms are the source of the negative results. We determine that *two* of Savage's axioms are incompatible with Paretian aggregation–P3 and P4. Our second contribution is to demonstrate the possibility of Paretian aggregation without P3 and P4 by providing a representation for social preference conforming to the state-dependent expected utility model.

6 Appendix–Proof of Theorem 1

To prove Theorem 1, we use the following Lemma.

Lemma 1: Suppose $(U_1, ..., U_m)$ is a vector of state-dependent expected utility functionals. Then

$$\{(U_1(f), ..., U_m(f)) : f \in \mathcal{F}\}$$

is convex.

Proof: We need to show that for all $f, g \in \mathcal{F}$, $\alpha \in [0, 1]$, there exists some $h \in \mathcal{F}$ such that for all $i = 1, ..., m$, $U_i(h) = \alpha U_i(f) + (1 - \alpha) U_i(g)$.

Therefore, let $f^*, g^* \in \mathcal{F}$, $\alpha^* \in [0, 1]$.

Let $X(f^*, g^*) \equiv (range(f^*)) \cup (range(g^*))$. As f^*, g^* are simple acts, $|X(f^*, g^*)| < \infty$. Let $\mathcal{F}(f^*, g^*) \subset \mathcal{F}$ be the set of acts whose range lies in $X(f^*, g^*)$.

By assumption, for all $i = 1, ..., m$, there exists $u_i : X \times S \to \mathbb{R}$ and μ_i on (S, Σ) such that $U_i(f) \equiv \int_S u_i(f(s), s)\, d\mu_i(s)$. For all $i = 1, ..., m$, and for all $x \in X(f^*, g^*)$, let ν_i^x be a measure on Σ defined by

$$\nu_i^x(E) \equiv \int_E u_i(x, s)\, d\mu_i(s).$$

Then for all $i \in N$, $x \in X(f^*, g^*)$, ν_i^x is countably additive, nonatomic, and finite.[5] By definition of the integral, for all $i \in N$, $f \in \mathcal{F}(f^*, g^*)$

$$U_i(f) = \sum_{x \in X(f^*, g^*)} \nu_i^x\left(f^{-1}(x)\right).$$

Let Π be the set of Σ-measurable ordered $X(f^*, g^*)$-partitions. Formally, Π is defined as the set of functions $P : X(f^*, g^*) \to \Sigma$ satisfying *i)* $\bigcup_{X(f^*, g^*)} P(x) = S$ and *ii)* for all $x, y \in X(f^*, g^*)$ such that $x \neq y$, $P(x) \cap P(y) = \emptyset$.

[5] To see that ν_i^x is nonatomic, suppose that it is not. Then there exists some $E \in \Sigma$ such that $\nu_i^x(E) > 0$, and for all $F \subset E$, $\nu_i^x(F) \in \{0, \nu_i^x(E)\}$. Let $\{E_m\}_{m=1}^{\infty}$ be a sequence such that $E_1 = E$, and for all m, $E_m \subset E_{m-1}$, $\mu_i(E_m) = \frac{1}{2}\mu_i(E_{m-1})$, and $\nu_i^x(E_m) = \nu_i^x(E)$. By countable additivity, $\mu_i\left(\bigcap_{m=1}^{\infty} E_m\right) = 0$ and $\nu_i^x\left(\bigcap_{m=1}^{\infty} E_m\right) = \nu_i^x(E) > 0$, a contradiction. Finiteness follows as $u_i(x, \cdot)$ is μ_i-integrable.

Clearly, there is a bijection ψ between $\mathcal{F}(f^*, g^*)$ and Π, given by for all $f \in \mathcal{F}(f^*, g^*)$ and for all $x \in X(f^*, g^*)$, $\psi(f)(x) \equiv f^{-1}(x)$. In particular,

$$A \equiv \left\{ \left(\nu_i^x \left(P\left(x\right)\right)\right)_{i,x} \subset \mathbb{R}^{m \times X(f^*, g^*)} : P \in \Pi \right\}$$

is equal to

$$B \equiv \left\{ \left(\nu_i^x \left(f^{-1}\left(x\right)\right)\right)_{i,x} \subset \mathbb{R}^{m \times X(f^*, g^*)} : f \in \mathcal{F}\left(f^*, g^*\right) \right\}.$$

By Theorems 1 and 4 of Dvoretzky, Wald, and Wolfowitz [7], it follows that A is convex.[6] Thus B is also convex.

By summing the columns of the elements of B, we obtain $\{(U_1(f), ..., U_m(f)) : f \in \mathcal{F}(f^*, g^*)\}$. Convexity is preserved under this summation. Lastly, note that $\{(U_i(f^*))_i, (U_i(g^*))_i\} \subset \{(U_1(f), ..., U_m(f)) : f \in \mathcal{F}(f^*, g^*)\}$. Thus, there exists some $h \in \mathcal{F}(f^*, g^*)$ such that for all $i = 1, ..., m$, $U_i(h) = \alpha^* U_i(f^*) + (1 - \alpha^*) U_i(g^*)$.

■

We now prove the theorem.

Proof: It is obvious that if there exists a vector $\lambda \in \mathbb{R}^N$ such that for all $f, g \in \mathcal{F}$, $f \succeq_0 g \Leftrightarrow \sum_N \lambda_i U_i(f) \geq \sum_N \lambda_i U_i(g)$, then $\succeq_0$ satisfies Pareto indifference with respect to $(\succeq_1, ..., \succeq_n)$. We will now show that it is a state-dependent subjective expected utility preference. If for all $f, g \in \mathcal{F}$, $f \sim_0 g$, then the claim is obvious. So, assume there exist $f', g' \in \mathcal{F}$ such that $f' \succ_0 g'$.

For all $i \in N$, U_i has two components, a utility index u_i, and a probability measure μ_i. Let $\mu \equiv \frac{\sum_N \mu_i}{|N|}$. Then μ is a probability measure defined on (S, Σ). For all $i \in N$, μ_i is absolutely continuous with respect to μ. By the Radon-Nikodym Theorem, for all $i \in N$, there exists a Σ-measurable, μ-integrable function $h_i : S \to \mathbb{R}$ such that for all $f \in \mathcal{F}$,

$$\int_S u_i\left(f\left(s\right)\right) d\mu_i\left(s\right) = \int_S u_i\left(f\left(s\right)\right) h_i\left(s\right) d\mu\left(s\right).$$

Thus, for all $f \in \mathcal{F}$,

$$\sum_N \lambda_i U_i\left(f\right) = \int_S \left(\sum_N \lambda_i u_i\left(f\left(s\right)\right) h_i\left(s\right)\right) d\mu\left(s\right).$$

[6] This follows as Theorems 1 and 4 of Dvoretzky, Wald, and Wolfowitz imply that $\left\{ \left(\nu_i^x \left(P\left(y\right)\right)\right)_{(i,x),y} \subset \mathbb{R}^{(m \times X(f^*, g^*)) \times X(f^*, g^*)} : P \in \Pi \right\}$ is convex. The set A is a projection of this set on the subspace in which $x = y$.

Let $u : X \times S \to \mathbb{R}$ be defined by

$$u(x,s) \equiv \sum_N \lambda_i u_i(x) h_i(s).$$

Thus, for all $x \in X$, $u(x,\cdot)$ is Σ-measurable and μ-integrable, and for all $f \in \mathcal{F}$,

$$\sum_N \lambda_i U_i(f) = \int_S u(f(s),s)\, d\mu(s).$$

Conversely, suppose that for all $i \in N$, $\succeq_i$ is a subjective expected utility preference. Suppose that $\succeq_0$ is a state-dependent subjective expected utility preference which satisfies Pareto indifference with respect to $(\succeq_1, ..., \succeq_n)$.

For all $i \in N$, let $U_i : \mathcal{F} \to \mathbb{R}$ be a subjective expected utility functional representing $\succeq_i$, and U_0 a state-dependent subjective expected utility functional representing $\succeq_0$. By Lemma 1 above and Proposition 1 of DeMeyer and Mongin [5], there exist $\lambda \in \mathbb{R}^N$ and $K \in \mathbb{R}$ such that for all $f \in \mathcal{F}$,

$$U_0(f) = K + \sum_N \lambda_i U_i(f).$$

Thus, for all $f, g \in \mathcal{F}$, $f \succeq_0 g$ if and only if $K + \sum_N \lambda_i U_i(f) \geq K + \sum_N \lambda_i U_i(g)$. Equivalently, $f \succeq_0 g$ if and only if $\sum_N \lambda_i U_i(f) \geq \sum_N \lambda_i U_i(g)$. ■

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