

# Profit Sharing (with workers) Facilitates Collusion (among firms)

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### **Abstract**

This paper shows how profit sharing by firms with workers always facilitates increased collusion among firms in a dynamic oligopoly environment with uncertain demand. Expected firm profits are increased both if worker wages are tied to market conditions, or if workers instead receive a share of firm profits. We first show that firm profits are always increased by tying wages to market conditions. The profit-maximizing agreement features only partial sharing because increased sharing raises the expected price-wage differential, but reduces price-wage variability. We then show that for any cartel size, there are always market conditions for which expected firm profits are increased simply by transferring some expected profit to workers, through the impact of this transfer on the incentive to cheat on the cartel.

# 1 Introduction

This paper addresses a basic contracting issue: How does the sharing of profits between a firm and its employees affect the ability of firms to support collusive oligopoly pricing and oligopoly profits? We explore this question in the dynamic oligopoly environment with uncertain demand first characterized by Rotemberg and Saloner (1986).

We obtain a stark answer: Profit sharing between firms and their employees *always* facilitates collusion between firms in the output market. Indeed, we show that expected profits can be raised by *giving* workers a share of firm profits, *without extracting a corresponding reduction in wages*.

It is important to emphasize that our findings do not revolve around either worker moral hazard or insurance explanations. In our barebones economy there is no problem eliciting worker effort, so that profit sharing is not required to overcome worker moral hazard. Further, all parties are risk-neutral, so there is no role for insurance.

Rather, the driving economic force is that profit sharing between workers and firms favorably alters the strategic interaction of firms in the output market. The economics underlying our results can be gleaned from recalling equilibrium outcomes in Rotemberg and Saloner (1986). There, firms repeatedly interact in an economy with independently and identically distributed demand shocks. Because expected future collusive profits do not depend on the period demand shock, but the gains from cheating on the cartel rise with demand, firms can support monopoly profits only if the demand shock is sufficiently low. If demand is too high, then total industry output is increased beyond the monopoly level just until firms cease to have an incentive to cheat on the cartel.

Now consider the impact of tying worker compensation to market conditions. Tying compensation effectively reduces worker compensation when demand is low and raises worker compensation when demand is high. But in low demand states firms do not have an incentive to cheat on the cartel, so that the lower worker compensation raises cartel profits in these low demand states. In contrast, in sufficiently high demand states, firm profit is completely unaffected by increased worker compensation, as profit is already constrained by the heightened incentive to cheat on the cartel.

The profit-maximizing worker-firm agreement generally features only partial profit sharing so that, along the equilibrium path, net firm profit rises monotonically with the demand realization. This is because while profit sharing increases the expected price-“wage” difference, it also reduces the variance of this difference. Because the period profit function is convex, increased profit sharing

eventually reduces this variance by enough that expected firm profits fall (unless monopoly profits cannot be sustained even for the worst demand realization). However, since the variance has only a second-order impact on firm profit, but raising the expected price-“wage” difference has a first-order impact, it follows that some profit sharing is always optimal.

We first consider an environment in which workers’ wages can be tied to market conditions in a linear fashion, so that wages are higher when market demand is higher. This framework highlights the tradeoff of increased profit sharing on (i) the expected price-wage difference, and (ii) the variance of the price-wage difference. In this environment, we show that there always exists an incentive-compatible wage agreement that allows firms to capture increased profit. We then relax the restriction that wages are linearly linked to market conditions and require only that the wage be non-decreasing in demand. We show that the resulting profit-maximizing wage-tying agreement takes the form of an option, with wages fixed at a low level unless demand is sufficiently high.

We then explicitly allow for profit sharing, so that employees can receive a share of firm profits. This reveals another benefit of profit sharing: If a firm cheats by increasing output and hence profits, not only must it pay each worker more because its profits are higher, but it must also hire and pay more workers to produce the output. Consequently, an explicit profit-sharing agreement with workers further reduces a firm’s incentive to cheat on a cartel. To highlight this, we provide sufficient conditions under which firms can increase expected profits simply by *giving* employees a share of firm profits, without demanding a lower wage in return. We show that for a cartel of any size, there exist market conditions such that giving workers a share of firm profits raises cartel profits; the reduction in the incentive to cheat on the cartel supported by this transfer to workers supports monopoly pricing in enough additional higher demand states to more than offset the transfer of surplus to employees.

Our work is related to two research strands. Many, many papers investigate collusion among firms in dynamic oligopoly environments. Haltiwanger and Harrington (1991) explore collusion in a deterministic variant of Rotemberg and Saloner in which demand moves cyclically over time. Green and Porter (1984), and Abreu et al. (1986, 1990) explore collusion when the realized price depends on both aggregate output and an unobserved shock so that firms cannot unravel from the price realization whether some firm ‘cheated’ on the cartel. Fudenberg, Levine and Maskin (1994), and Kandori and Matsushima (1998), provide versions of the folk theorem in such environments. Other papers that explore aspects of collusion in environments where firms repeatedly interact include Fershtman and Pakes (2000), Eaton and Eswaran (1997, 2001) and Lambson (1987, 1994).

A second research strand explores how the incentive structures that a firm provides managers affect the strategic interaction of firms in the output market. For example, Fershtman and Judd (1987) and Sklivas (1987) show that if firms compete in quantities, then compensating a manager for sales serves to commit the firm to producing more output, which, in turn, induces competitors to reduce output. Conversely, if firms compete in prices, then firms can raise profits by designing incentives that discourage under-cutting by punishing excessive sales. Reitman (1993) extends their analysis to allow firms to compensate managers with stock options. Basu (1995) also builds on these works, explicitly modelling the decision of whether or not to hire managers. Barcena-Ruiz and Paz Espinoza (1996) investigate the temporal aspect of manager contracts in such a model.

In a similar vein, several papers explore the interaction between product market competition and capital markets. Brander and Lewis (1986, 1988) show how a firm's financial structure affects the nature of competition in output markets. For example, when demand is uncertain, debt commits a firm to producing more output (because firms repay loans only when demand is high). Bolton and Scharfstein (1990), Maksimovic (1998) and Faure-Grimaud (2000) deal with related topics. Maksimovic and Titman (1991) explore financial structure and reputation for product quality.

The focus of this second research strand is very different from ours—addressing in static contexts how managerial incentives or financial structure can be designed to commit the firm to being aggressive or passive in the output market, as desired. In contrast, our focus is on how managerial compensation in the form of profit sharing affects the dynamic incentives of a firm to cheat on a cartel, and hence the ability of the cartel to support more profitable collusion when demand is uncertain.

The paper closest in spirit to ours is Spagnolo (2000). Spagnolo shows that output-deciding managers can collude at higher levels if compensation is given in stocks. However, very different economic forces are at work in his model. Spagnolo studies a market with no uncertainty, and explicitly considers a stock market, where the value of a stock is simply its expected sum of discounted payoffs. Therefore, managers' *period* payoffs are directly tied to the sum of discounted firm profits. Thus, when deviating from collusive strategies, managers incur a period loss through the loss of future profits. In this sense, stock-based compensation directly facilitates collusion.

## 2 The Model

The basic framework is that of Rotemberg and Saloner (1986):  $n$  firms producing a homogeneous good repeatedly interact in an infinite horizon economy with stochastic demand. Period market

demand at time  $t$  is given by

$$D_t(\theta_t, P_t) = \theta_t - P_t,$$

where  $\theta_t$  is an independently and identically distributed demand shock, with distribution function  $F$  on its positive support  $[\underline{\theta}, \bar{\theta}]$ .<sup>1</sup> The risk-neutral firms share common discount factor  $\beta \in (0, 1)$ , and compete in prices in the output market.

To investigate the impact of profit sharing between workers and firms, we must model the production function of a firm more explicitly than do Rotemberg and Saloner. We assume that one hour of labor is required to produce one unit of the good. Workers are risk-neutral, and have an outside opportunity of  $\bar{w}$  per hour, which, without loss of generality, we normalize to zero.

Absent profit sharing, our framework corresponds to Rotemberg and Saloner. Firms support maximal period profits by threatening to revert to marginal cost pricing if any firm ever deviates from the collusive pricing agreement. Let  $\Pi^m(\theta) = \left(\frac{\theta - \bar{w}}{2}\right)^2$  denote monopoly profits if the demand shock is  $\theta$ , and let  $V$  represent expected period profit per firm along the equilibrium path in which firms co-operate. Along a co-operative path, firms split total profit equally. Given a demand shock  $\theta$ , monopoly profits are sustainable if and only if

$$\left(\frac{\Pi^m(\theta)}{n}\right) + \left(\frac{\beta}{1-\beta}\right)V \geq \Pi^m(\theta).$$

That is, incentive compatibility mandates that the expected profits from continued cooperation (the left-hand side), exceed the one-time gain from cheating on the cartel by lowering price slightly to capture the entire market profit (the right-hand side). The following proposition formalizes this intuition, where  $\Pi^s(\theta)$  is sustainable profit in state  $\theta$ .

**Proposition 1** (*Rotemberg and Saloner*) *Suppose that monopoly profits can be sustained in some demand states, but not in others. That is, there exists a  $\theta^* \in (\underline{\theta}, \bar{\theta})$  which solves*

$$\left(\frac{\beta}{1-\beta}\right) \left[ \int_{\underline{\theta}}^{\theta^*} \Pi^m(\theta) F(d\theta) + (1 - F(\theta^*)) \Pi^m(\theta^*) \right] = (n-1) \Pi^m(\theta^*).$$

*Then profit-maximizing pricing strategies support monopoly profits in demand states  $\theta \leq \theta^*$ , but only support profits of  $\Pi^m(\theta^*)$  in higher demand states  $\theta > \theta^*$ :*

$$\Pi^s(\theta) = \begin{cases} \Pi^m(\theta), & \theta \leq \theta^* \\ \Pi^m(\theta^*), & \theta > \theta^* \end{cases}.$$

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<sup>1</sup>None of our results qualitatively depend on the assumption that demand shocks are i.i.d. Demand could follow a Markov process, or there could be cycles in demand as in Haltiwanger and Harrington (1991).

In particular, along the equilibrium path, in a low demand state  $\theta \leq \theta^*$ , firms set the monopoly price of  $\frac{\theta}{2}$ , and each firm produces  $\frac{\theta}{2n}$ . If demand is higher,  $\theta > \theta^*$ , the monopoly price cannot be sustained, as firms would have an incentive to undercut it to garner the entire market profit.

## 2.1 Profit sharing: Demand-linked wage contracts

We now explore how outcomes are affected when worker compensation is tied to market conditions. Each period, before a demand shock is realized, firms and workers sign binding wage agreements in which the wage depends on the demand realization. One may think of a union bargaining for an agreed-upon industry wage. A demand-contingent wage is agreed to, and this wage is legally binding. The wage contracts are publicly observed before firms make output choices.

We first consider linear wage contracts (section 2.3 considers more general wage contracts), so that the period wage firm  $i$  pays its workers when demand is  $\theta_t$  is

$$\omega(\theta_t, a_t^i) = \omega(a_t^i) + a_t^i(\theta_t - E(\theta)).$$

Here,  $a_t^i$  measures the degree to which wages are tied to the demand realization, and  $\omega(a_t^i)$  is a fixed component of compensation that does not depend on demand. It eases presentation to have  $a_t^i$  weight the difference between current and expected demand, and this is without loss of generality.

We index  $\omega(a_t^i)$  by  $a_t^i$  because incentive compatibility will imply a linkage between fixed and tied components of compensation. That is, the wage contract must provide workers at least their reservation alternative in expectation: firms must ensure that it is incentive compatible for workers to sign the contract. In particular, it must be the case that

$$\int_{\underline{\theta}}^{\bar{\theta}} (\omega(a_t^i) + a_t^i(\theta - E(\theta))) Q(\theta, a_t^i) F(d\theta) \geq \int_{\underline{\theta}}^{\bar{\theta}} \bar{\omega} Q(\theta, a_t^i) F(d\theta).$$

In this inequality, we have subtracted from each side the payoff to a worker for hours not worked (for which the worker obtains  $\bar{\omega}$  per hour). Thus,  $Q(\theta, a_t^i)$  is a function that details the total hours of work when the demand realization is  $\theta$  and the wage contract parameter is  $a_t^i$ . The left-hand side of the inequality is what workers expect from working with this wage contract; the right-hand side is what they get if they use their labor to obtain their outside opportunity. In equilibrium, this constraint holds as an equality as firms will never set  $\omega(a)$  any higher than necessary to induce workers to work. If  $a_t^i = 0$ , then the standard fixed-wage outcome obtains,  $\omega(\theta_t, a_t^i) = \bar{\omega}$ . Suppose that, instead, firms partially tie wages to demand, i.e.,  $a_t^i \in (0, 1)$ . Then because firms produce more output when demand is high, and workers are paid more in those high demand states, it

follows that the firm can lower the fixed component of wages,  $\omega(a_t^i)$ , and still retain the incentive compatibility of the wage contract. Finally, if  $a_t^i = 1$ , a firm is ‘fully-insured’ against the demand shock, as  $\theta_t - \omega(\theta_t, 1) = E(\theta) - \omega(1)$ . As a result, output in a symmetric equilibrium in which all firms choose  $a_t^i = 1$  does not vary with market conditions, so that  $\omega(\theta_t, 1) = \bar{\omega}$ .

After the wage contract is signed, the period demand shock is realized. After observing the demand shock, firms simultaneously decide on prices given the history of past demand shocks, wage contracts and prices. That is, a firm’s strategy is a sequence of functions  $\{(a_t^i, P_t^i)\}$  where  $a_t^i$  maps histories of length  $t - 1$  into wage contracts, and  $P_t^i$  maps from histories of length  $t - 1$  together with  $(a_t, \theta_t)$  into a price choice. The price choices by firms determine a firm’s market demand, and the firm pays the contractually-determined wage to workers to produce this quantity.

The assumption that contracts are binding for workers is made only for simplicity; our findings extend when binding contracts cannot be signed (i.e., so that workers would quit if their reservation alternative dominates the wage offer). When binding contracts can be signed, because firms extract all surplus each period in wage negotiations, so that a worker’s ex-ante incentive-compatibility constraint holds at equality. Then, ex post, if  $\theta_t$  is low, workers regret signing the wage contract. However, to retrieve ex-post incentive compatibility for workers even in the lowest demand state  $\underline{\theta}$ , firms only need to raise the fixed wage component  $w(a_t^i)$  slightly, and threaten never to hire a worker who rejects a wage offer. Section 2.2 offers a more complete characterization of ex-post incentive-compatible wage contracts. Note that firms do not have a similar incentive to chisel workers in high demand states, even were wage contracts not legally binding on firms. That is, the ex-post incentive-compatibility constraint for firms does not bind because firms anticipate surplus from future collusion, and hence have something to lose from renegeing. Most transparently, we can assume that if a firm reneges on a wage agreement and pays a worker just his outside option, this is observed and firms cease to collude (and, indeed, we can assume that no worker ever works for such a firm in the future).

Our analysis focuses on symmetric equilibrium paths in which firms produce the same output and split profits equally. From now on, without loss of generality, we drop the index  $i$  from strategies. We look at firm trigger strategies in which if any firm has deviated in the past from the specified symmetric wage contract and pricing choices, then each firm sets price equal to marginal cost in the future. Such strategies support maximal firm profits.

Along a symmetric action path, period market profit is given by

$$\Pi_t(\theta_t, P_t) = (\theta_t - P_t) [P_t - [\omega(a_t) + a_t(\theta_t - E(\theta))]].$$



Firms choose the wage contract parameter,  $a_t$ , and period price function  $P(a_t, \theta_t)$ , to maximize the sum of discounted expected profits subject to the constraint that the wage contract be incentive compatible for workers. Because the optimal wage contract parameter does not vary with time, we denote it simply by  $a$ , and let

$$\Pi^m(\theta, a) = \left( \frac{\theta + a(E(\theta) - \theta) - \omega(a)}{2} \right)^2$$

denote monopoly profits as a function of demand shock and wage contract parameter,  $a$ .

As in Rotemberg and Saloner, the expected profit-maximizing strategy profile along the equilibrium path features a pricing schedule with a critical demand level,  $\theta^*(a)$ , such that each firm sets price equal to the monopoly price if  $\theta \leq \theta^*(a)$ , and prices so that period profits just equal  $\Pi^m(\theta^*(a), a)$  if  $\theta > \theta^*(a)$ . Off the equilibrium path, firms set prices that generate zero profit.

Denoting these sustainable profits as a function of the demand shock and wage contract by  $\Pi^s(\theta, a)$ , we have

$$\Pi^s(\theta, a) = \begin{cases} \Pi^m(\theta, a), & \theta \leq \theta^*(a) \\ \Pi^m(\theta^*(a), a), & \theta > \theta^*(a) \end{cases}.$$

Therefore, the expected profits for any given firm under these strategies are:

$$(1/n) \sum_{t=0}^{\infty} \beta^t E[\Pi^s(\theta, a)].$$

The equilibrium level of  $a^*$  maximizes these profits (where the worker incentive compatibility constraint is now implicit). Our first proposition shows that as long as firms cannot support monopoly profits in the Rotemberg and Saloner environment in *every* demand state, then it is always beneficial for firms to tie worker compensation to market conditions:

**Theorem 1** *If  $\theta^*(0) < \bar{\theta}$ , so that monopoly profits cannot be supported in every demand state, then firms optimally tie worker compensation to the demand realization, setting  $a^* > 0$ .*

All proofs are in the appendix. ■

Because the optimal linear wage contract features tied wages, an immediate corollary is that the unconstrained profit-maximizing wage contract also features wages that are tied to demand.

To understand Theorem 1, consider a version of the model in which firms do not extract lower fixed wage concessions in return for demand-sensitive wages, so that the wage when demand is  $\theta$

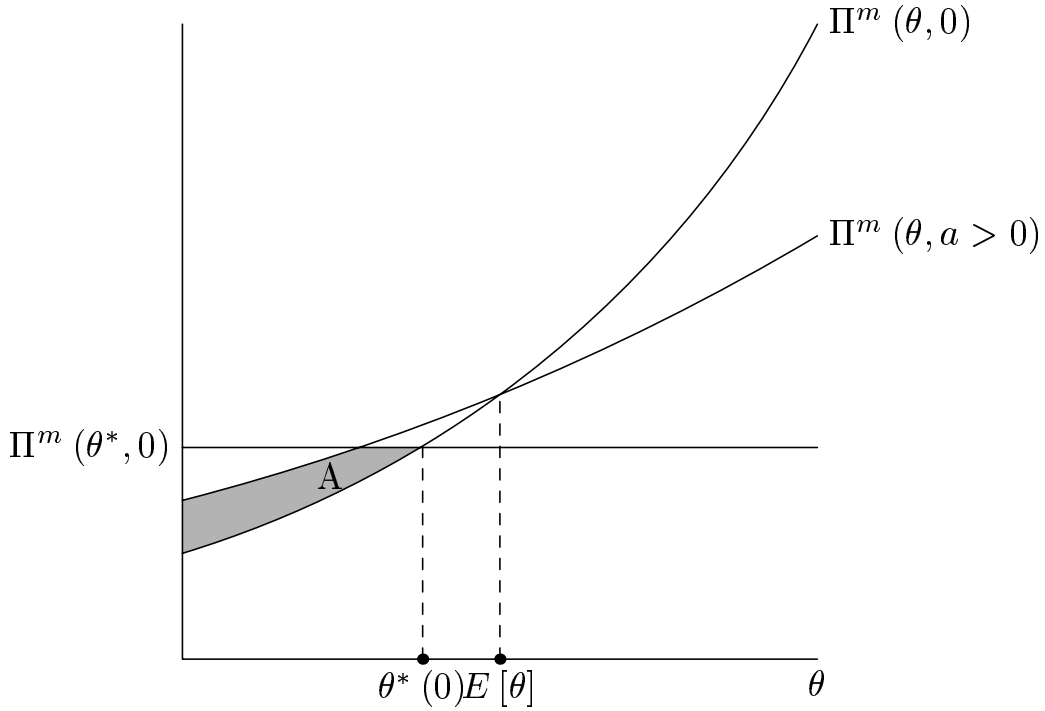


Figure 1: Change in Expected Profits when  $\theta^*(0) < E[\theta]$ .

is  $\bar{w} + a(\theta - E[\theta])$ . Note that with this wage, expected worker compensation exceeds  $\bar{w}$  because both output and wage rise with  $\theta$ . That is, because firms produce more when demand is higher, such wage contracts transfer some of the surplus generated by increased collusion to workers, reducing the profitability of tying wages to market conditions. Figure 1 depicts a situation in which continuation payoffs are fixed. The figure contrasts one period monopoly profits when firms do not tie wages to market conditions,  $\Pi^m(\theta, 0)$ , with monopoly profits when wages are tied,  $\Pi^m(\theta, a > 0)$ . The two profit levels are equal if  $\theta = E[\theta]$ , so that  $\bar{w} + a(\theta - E[\theta]) = \bar{w}$ . If  $\theta < \theta^*(0)$ , then  $\Pi^m(\theta, a > 0) > \Pi^m(\theta, a = 0)$ , and if  $\theta > \theta^*$ , then  $\Pi^m(\theta, a > 0) < \Pi^m(\theta, a = 0)$ . This figure is drawn so that if wages are not tied to market conditions, then the maximal demand state that supports monopoly profits,  $\theta^*(0)$ , is less than  $E[\theta]$ . This implies that for demand shocks exceeding  $\theta^*(0)$ , including  $\theta = E[\theta]$ , maximal sustainable profits are just  $\Pi(\theta^*(0), 0)$ . However, by setting  $a > 0$  and using the same continuation payoffs, greater collusive profits are supported for all  $\theta < \theta^*(0)$ , with *no* reduction in period profits for  $\theta > \theta^*(0)$ . Thus, by setting  $a > 0$ , firms gain in the region  $\theta < \theta^*(0)$ , but there are no corresponding losses in the region  $\theta > \theta^*(0)$ . The profit increase from tying wages to market conditions corresponds to the shaded area  $A$  in the diagram.

Further, since expected period one profits are increased, the same analysis applies to future periods; continuation payoffs must also be greater. Therefore, by setting  $a > 0$ , firms can support

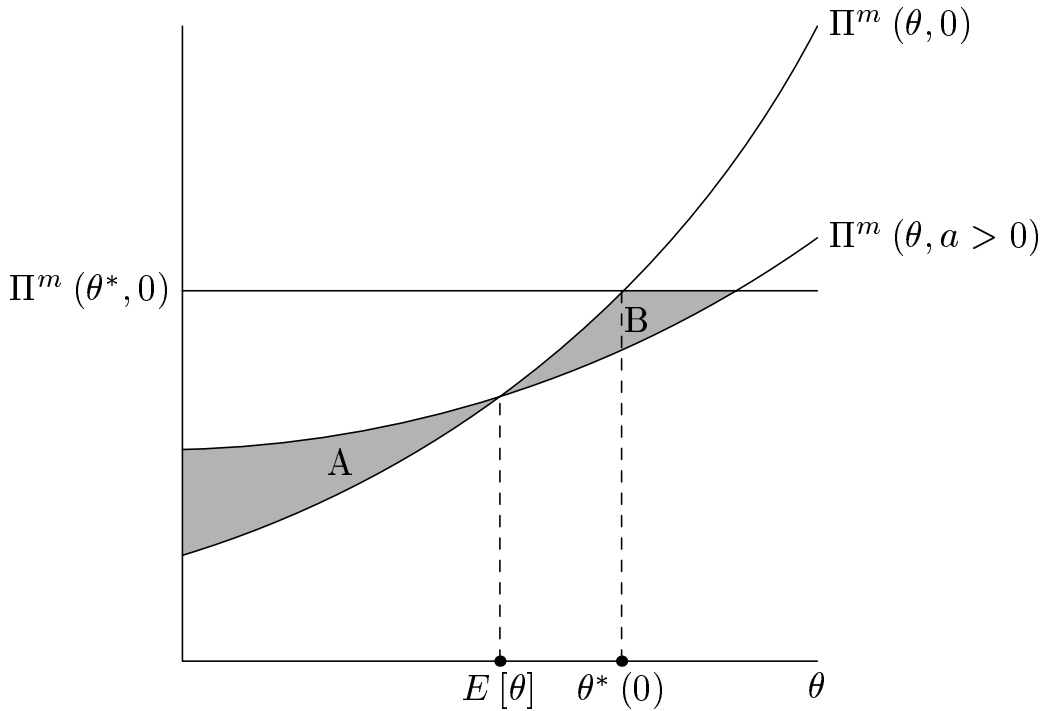


Figure 2: Change in Expected Profits when  $\theta^*(0) > E[\theta]$ .

more collusive pricing, even if  $\theta > \theta^*(0)$ . Indeed, the reason that tying wages perfectly to market conditions, i.e., setting  $a = 1$ , is not optimal is that eventually, monopoly profits can be supported for demand realizations  $\theta > E[\theta]$ , creating a cost to tying wages to market conditions.

Figure 2 illustrates this second case, where  $\theta^*(0) > E(\theta)$ . Here, by tying wages to demand, firms still gain profit in low demand states (shaded area  $A$ ), but in contrast to Figure 1 lose profit in high demand states (shaded area  $B$ ). If  $B > A$ , then it is better not to tie wages to market conditions at all than to tie them to the degree that  $B > A$ .

What the proof of theorem 1 does is show that when firms extract all surplus in contracting from workers, expected firm profits can always be increased by *slightly* tying wages to market conditions, i.e., by choosing an  $a > 0$  that is sufficiently small. Because  $w(a) < \bar{w}$ , the expected price-wage differential is greater when wages are tied to the demand realization. However, the variance of the price-wage differential is reduced. Reducing this variance reduces expected profits because the profit function is convex. Nonetheless, because the increase in the mean of the price-wage differential has a first-order impact on expected profits, while the reduction in its variance has only a second-order impact, it follows that at the very least, some tying of wages to market conditions always raises payoffs.

Formally, what we show in the appendix is that

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)]|_{a=0} = \int_{\underline{\theta}}^{\theta^*(0)} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) \quad (1)$$

$$> \int_{\underline{\theta}}^{\bar{\theta}} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) \quad (2)$$

$$> \frac{1}{2} \int_{\underline{\theta}}^{\bar{\theta}} \theta \left( \frac{E(\theta) - \theta - \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta)(\theta/2) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) F(d\theta)}}{2} \right) F(d\theta) = 0, \quad (3)$$

where  $\Pi^s(\theta, a)$  is the sustainable level of one period profit when after that period, all firms use the Rotemberg and Saloner equilibrium ( $a = 0$ ).

In general, it may be optimal to tie wages far more significantly to market conditions. Figure 3 illustrates how the optimal degree to which wages should be tied to market conditions varies with  $\theta^*(0)$ , the highest demand realization for which monopoly profits can be supported given  $a = 0$ . Note that  $\theta^*(0)$ , which rises with the discount factor  $\beta$  and falls with number of firms,  $n$ , is a sufficient statistic for the primitives describing the economy. Figure 3 illustrates the profit-maximizing wage-tying rule when the demand shock is uniformly distributed on  $[1, 10]$ . As collusion becomes sustainable in more demand states, the degree to which wages should be tied to market conditions falls. This is because a cartel that can sustain monopoly profits in very high demand states has more to lose by signing wage tying contracts with workers than a cartel that cannot. Observe that perfectly tying wages to market conditions is optimal only when in the standard Rotemberg and Saloner environment in which wages are not tied to market conditions, the cartel cannot sustain monopoly profits even in the worst demand state.

## 2.2 Relaxing the assumption that wage contracts are binding

We now return to our earlier discussion regarding the robustness of our results to relaxing the assumption that wage contracts are binding. Suppose that workers are infinitely lived, with common discount factor  $\gamma \in (0, 1)$ , and that now, *after* the demand shock is realized, workers can decide whether or not to abide by the wage agreement. If a worker ever refuses to work at the contractually-determined wage, firms can find replacement workers in the future, so that the worker can only obtain his reservation payoff thereafter. Firms must now augment the fixed wage component to  $\omega(a) + \epsilon(a)$ , where  $\epsilon(a)$  provides workers just enough expected surplus from the future worker-firm relationship to ensure that it is incentive compatible ex post for workers to supply their labor. Ex-post incentive compatibility requires that the workers willingly accept the wage contract even

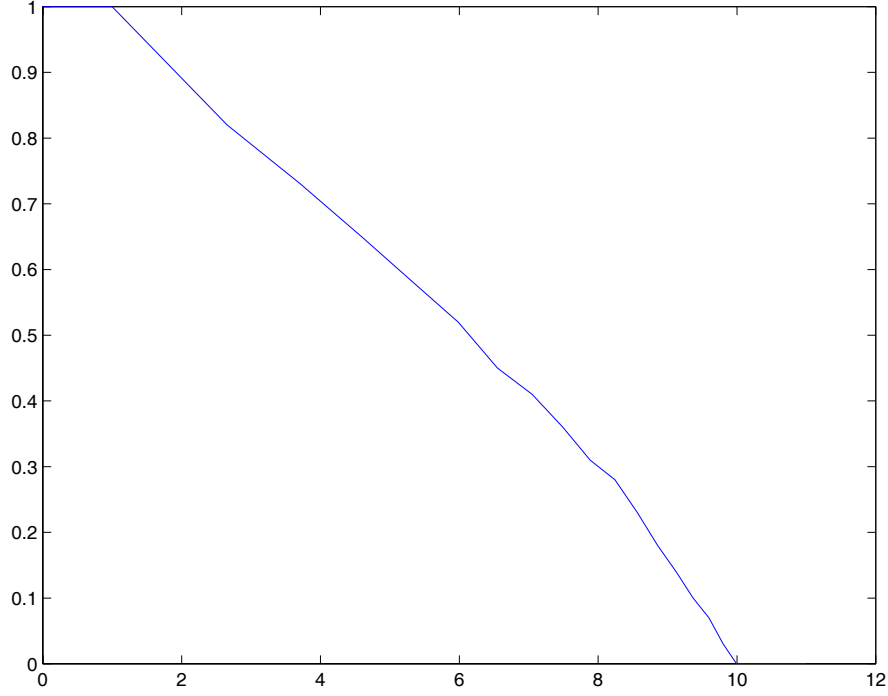


Figure 3: Optimal wage-tying parameter  $a$  as a function of  $\theta^*(0)$

in the worst demand state,  $\underline{\theta}$ ,

$$\left[ \omega(a) + \epsilon(a) + a(\underline{\theta} - E(\theta)) \right] \left( \frac{\underline{\theta}}{2} \right) + \left( \frac{\gamma}{1 - \gamma} \right) \int_{\underline{\theta}}^{\bar{\theta}} (\omega(a) + \epsilon(a) + a(\theta - E(\theta))) Q(\theta, a) F(d\theta) \geq 0,$$

where the right-hand side reflects the normalization,  $\bar{\omega} = 0$ . For any given  $\epsilon(a) > 0$ , this relationship is satisfied for all  $\gamma$  sufficiently close to one. Indeed, if workers are arbitrarily patient, firms can obtain ex-ante incentive compatibility of workers by offering a vanishingly small fixed wage increase,  $\epsilon(a) \rightarrow 0$ . In turn, this implies that our analysis extends immediately. More generally, if workers are more impatient, firms must offer workers a greater share of the surplus from tying wages to market conditions. In this case, tying wages to market conditions raises expected collusive firm profits only if, in the absence of tying wages, it is sufficiently difficult to support monopoly profits, i.e., if  $\theta^*(0)$  is sufficiently small relative to the other parameters characterizing the economy.

### 2.3 Characterizing the Optimal Non-linear Wage Contract

We have shown that *partially* tying wages to demand is *always* optimal, but fully tying wages is not. We established these results by working with a restricted domain of linear wage contracts that includes both contracts featuring no tying, and contracts featuring full tying. This raises the

question, “What are the properties of the optimal wage contract if we only impose the requirement that the wage paid be non-decreasing in  $\theta$ ?”

To answer this question, we work backwards. Given a function  $\omega : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$  that specifies the wage paid to workers when the demand shock is  $\theta$ , we calculate the profit-maximizing sustainable pricing strategies for firms. As in Rotemberg and Saloner, such strategies feature a profit level  $K(\omega)$  above which monopoly profits are not sustainable. That is, demand can be partitioned into two regions, a low demand region  $L$  and a high demand region  $H$ . If  $\theta \in L$ , firms set the monopoly price. If, instead, demand is higher,  $\theta \in H$ , the equilibrium price generates an aggregate profit of  $K(\omega)$ .

Let  $\omega^*(\theta)$  be the optimal monotone wage-tying rule as a function of  $\theta$ . Denote monopoly profits for wage-profile  $\omega^*$  as  $\Pi^m(\theta, \omega^*(\theta))$ . There exists some  $\theta^*$  such that for all  $\theta \leq \theta^*$ , monopoly profits are sustainable and for so that for all  $\theta > \theta^*$ , the maximal sustainable profits are  $K(\omega^*) = \Pi^m(\theta^*, \omega^*(\theta^*))$ . The following proposition describes the optimal wage-tying rule.

**Theorem 2** *The optimal wage-tying rule takes the form of an option. For all  $\theta \leq \theta^*$ , workers receive the same low wage,  $\omega^*(\theta) = \underline{w} < 0$ . For  $\theta > \theta^*$ ,  $\omega^*(\theta) > \underline{w}$  is strictly increasing in  $\theta$  and solves the following implicit equation:*

$$(\theta - 2\omega^*(\theta))(\sqrt{(\omega^*(\theta) - \theta)^2 - 4K(\omega^*)}) + (\omega^*(\theta) - \theta)^2 - 4K(\omega^*) + \omega^*(\theta)(\omega^*(\theta) - \theta) = 0. \quad (4)$$

*The value of  $\underline{w}$  gives workers zero surplus: the low wage bill when  $\underline{w}$  is set for  $\theta \in L$  is just offset by the high wage wage bill described above when  $\theta \in H$ . Thus,  $\underline{w}$  solves*

$$\int_L \underline{w} \left( \frac{\theta - \underline{w}}{2} \right) F(d\theta) = - \int_H \left( \frac{\theta - \omega(\theta) + \sqrt{(\theta - \omega(\theta))^2 - 4K(\omega)}}{2} \right) \omega(\theta) F(d\theta).$$

*Finally, at  $\theta^*$ , firms are indifferent cheating on the cartel and not:  $\left( \frac{\beta}{1-\beta} \right) V = \left( \frac{n-1}{n} \right) \Pi^m(\theta^*, \underline{w})$ .*

The intuition underlying the optimal wage contract is simple. For  $\theta \in H$ , monopoly pricing is not sustainable, so that firms price to realize an aggregate aggregate profit of  $K(\omega)$  in each state  $\theta$ . Because profits in these states is determined by the incentive to defect on the cartel and not wages, it follows that firms want to maximize the total industry wage bill in states  $\theta \in H$ . Equation (4) is essentially the first-order condition to this optimization problem. In states  $\theta \in L$ , firms can support monopoly profits. Hence, the optimal wage in states  $\theta \in L$  maximizes expected monopoly profits subject to the constraints that the low wages paid in states  $\theta \in L$  are just offset by the high wages paid in states  $\theta \in H$  (reflecting the normalization  $\bar{\omega} = 0$ ), and the requirement that wages

not fall with  $\theta$ . This monotonicity constraint binds—absent this constraint, the solution would be a wage that declines linearly with  $\theta \in L$ . Intuitively, monopoly profits are increased in region  $L$  by raising the variance of the margin  $\theta - \omega(\theta)$ . The constraint that wages be non-decreasing then implies that the optimal wage contract takes the form of an option with a constant wage for  $\theta \in L$  and a higher wage for higher demand realizations  $\theta \in H$ .

## 2.4 Pure Profit Sharing

It is straightforward to show that if, rather than tying wages to market conditions, firms share profits and extract the lowest incentive compatible fixed wage in return, then it is always optimal for firms to share profits with their workers. This leads us to pose a more ambitious question, asking under which conditions can firms raise profits by *giving* a share of profits to workers, *without demanding any wage concessions*? The structure of the economy is as before, save that we now assume that one worker produces one unit of the good. The period  $t$  compensation given to each worker when the demand shock is  $\theta_t$  is

$$\bar{w} + a_t \Pi_t(\theta_t),$$

where  $\Pi_t = (\theta_t - P_t)(P_t - \bar{w})$  and  $a_t$  is the profit-sharing rule. Incentive compatibility for workers is not an issue, as period profits are always positive. We again normalize  $\bar{w}$  to zero.

We study symmetric equilibria in which for any  $a$ , firms set the monopoly pricing whenever sustainable. To support such pricing, after a deviation by any firm, firms use marginal cost pricing. Then in demand state  $\theta$ , profits  $\Pi^*(\theta)$  are sustainable given profit-sharing  $a$  if and only if

$$\frac{(1 - aL(\theta, a)) \Pi^*(\theta)}{n} + \left( \frac{\beta}{1 - \beta} \right) V \geq (1 - anL(\theta, a)) \Pi^*(\theta),$$

where  $V$  is the expected period profit along the equilibrium path from colluding. Here,  $L(\theta, a)$  is the total number of workers needed per firm to produce the associated output. The left-hand side of this inequality represents expected discounted profits from continued collusion. Profits per firm are  $\Pi^*(\theta)/n$ , but each firm gives a share of  $aL(\theta, a)$  away to workers, according to the profit-sharing agreement. Continued cooperation in the current period implies that the expected discounted future payoffs are  $\frac{\beta}{1-\beta}V$ .

A firm that deviates from the cartel agreement optimally does so by marginally undercutting the cartel price. If a firm deviates it can obtain as much of the entire market as it desires. If the deviating firm meets the entire market demand, it can earn a profit of  $\Pi^*(\theta)$ , but it must return a

share  $anL(\theta, a)$  to its employees. Because the firm must hire more workers to produce the entire market output, not only must each worker be paid more if it cheats on the cartel, but the firm must also pay *more* workers. This reduces the attraction of cheating on the cartel. It is for this reason that simply signing contracts that give workers a share of firm profits can raise expected cartel profits, a form of ‘addition by subtraction’.

The following result obtains.

**Theorem 3** *Let period demand be uniformly distributed on  $[\underline{\theta}, \bar{\theta}]$ .<sup>2</sup> Let  $\theta^*(0)$  be the maximal demand shock such that monopoly profits can be supported in the absence of profit sharing (i.e., in the Rotemberg and Saloner environment). Then*

1. *For any  $(\underline{\theta}, \bar{\theta}, \beta)$ , there exists a  $n(\underline{\theta}, \bar{\theta}, \beta)$  such that if there are enough firms in the cartel, i.e.,  $n \geq n(\underline{\theta}, \bar{\theta}, \beta)$ , then profit sharing with workers raises profits.*
2. *For any  $n$ , given  $\underline{\theta}$ , if  $\theta^*(0) - \underline{\theta}$  and  $\bar{\theta} - \underline{\theta}$  are sufficiently small, firms can raise expected cartel profits by giving workers a share of profits.*
3. *For any  $(\underline{\theta}, n)$ , there exists a  $\bar{\theta}(\underline{\theta}, n)$  such that for any  $\bar{\theta} < \bar{\theta}(\underline{\theta}, n)$  there exists a  $\beta^*(\underline{\theta}, \bar{\theta}, n) < 1$  such that for any  $\beta \leq \beta^*(\underline{\theta}, \bar{\theta}, n)$ , profit sharing with workers raises profits.*

The appendix provides the precise condition characterizing when firms can support greater expected cartel profits by giving workers a small share  $a$  of profits, rather than setting  $a = 0$  is given in the appendix. In the neighborhood of  $a = 0$ , firms optimally set price equal to the monopoly price whenever the monopoly price can be supported, and if a firm decides to deviate from the cartel agreement, it optimally supplies the entire market. For larger profit-sharing rules, firms price in excess of the monopoly price whenever possible, choosing price to maximize the *residual* profits retained by firms,  $(1 - aL)(\theta - L)L$ . While raising price above the monopoly level reduces total profit, it also reduces the number of workers hired, which raises the net profits that cartel members retain. For similar reasons, if  $a$  is substantially greater than zero, a deviating firm would choose to ration some of the market, rather than hire the number of workers necessary to meet the entire market. Thus, Theorem 3 provides a sufficient, but not necessary condition, for net cartel profits to be raised by *giving* workers a share of profits.

The intuition underlying why profit-sharing becomes more attractive as the number of firms,  $n$ , in the cartel rises is that  $\theta^*(0)$  decreases, so that the gain to sharing profits is larger. Moreover, each

---

<sup>2</sup>The uniform assumption is made only to simplify the characterization.



firm employs fewer workers so the profit that each firm returns to its workforce declines, while a deviating firm must return the same entire market profit back to its workforce, independently of  $n$ . Figures 4-6 show how the marginal impact on expected period profits induced by sharing a marginal amount of profits with workers varies with  $\theta^*$  (0) for  $n \in \{2, 4, 10\}$  when the demand shock is uniformly distributed on  $[0, 10]$ . We see that (a) if there are two firms, profit sharing is never optimal; (b) if there are four firms, profit sharing is only optimal for intermediate values of  $\theta^*$ ; and (c) if there are ten or more firms, profit sharing raises expected cartel profits unless  $\theta^*$  is very large, in which case period monopoly profits can almost always be supported even in the absence of profit sharing.

### 3 Interpreting our results and relating our findings to the data

We have followed convention in the oligopoly literature and set up a model in which demand is stochastic, but worker productivity is constant. In sharp contrast, the real business cycle literature in macroeconomics focuses on non-stochastic demand (i.e., non-stochastic consumer preferences), but allows for productivity shocks. It is easy to show that identical analytical characterizations obtain if we instead assume that demand is constant, but the marginal productivity of a worker is stochastic. This is because what enters a firm's optimization problem is the difference between the demand shock and marginal cost. In both cases, it is optimal to tie wages to this difference. In both cases, wages rise when output is high.

While the analysis is the same, one key qualitative difference obtains. When demand is stochastic, wages are higher when demand shocks are higher: prices, wages, price-wage differential and output all vary procyclically with demand. In very sharp contrast, if instead, demand does not vary, but a worker's productivity is uncertain, then wages are higher when a worker's marginal product is higher, and hence output is greater, which implies that price is lower. Thus, when there is technological uncertainty, wages are procyclical, but prices are countercyclical, and hence price-wage differentials are counter-cyclical. Empirically, it has been well-established that precisely these counter-cyclical patterns hold in the data (see e.g., Kydland and Prescott (1990)).

There is also substantial empirical support tying worker compensation to market conditions—it is standard for workers to be compensated with equity. In the context of our model, the empirical questions become: Is profit sharing greater in more collusive industries? Is worker compensation tied more tightly to profit margins in such industries? Empirical research has not focused on these questions directly. There is evidence that more collusive industries tend to be more unionized, and unions tend to negotiate more wage tying. It has also been established that there is more

Figure 4:  $n = 2$

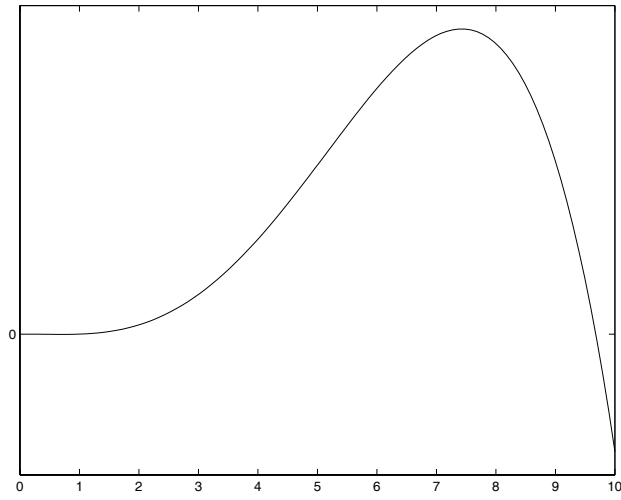
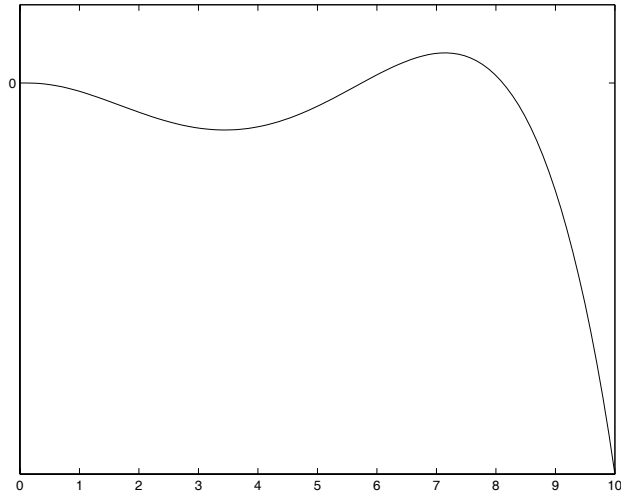
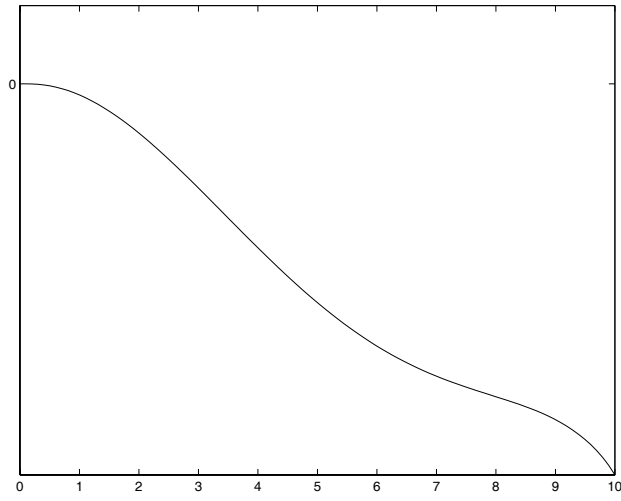


Figure 5:  $n = 10$

profit sharing in industries where profits are higher. For example, when industry profits are higher, workers in the industry earn higher wages than in comparable non-industry jobs. What has not been investigated thoroughly is the extent to which worker compensation in such industries varies procyclically with the profit margin. Finally, there is anecdotal evidence. For example, investment banking appears to be a collusive industry that supports high profits, and employee compensation is tightly tied to market conditions through the large annual bonus pools.

## 4 Conclusion

This paper offers an explanation for the prevalence of profit-sharing agreements between workers and firms. Importantly, our explanation revolves around neither moral hazard, nor risk sharing. Rather, we show that profit-sharing agreements between workers and firms facilitate collusion among firms in the output market. We highlight two features of profit-sharing agreements: (a) profit sharing ties worker compensation to market conditions, so that worker compensation is higher when demand is higher, and (b) if, to cheat on the cartel, a firm must hire more workers, then the firm must return a greater portion of those profits back to workers, reducing the gain to cheating.

We show that it is always optimal to tie worker compensation to market conditions. Tying compensation raises the expected price-wage differential, but reduces its variance. Raising the expected price-wage differential has a first order impact on expected profits, while reducing its variance has only a second order impact on profits. This immediately implies that it is always optimal to tie compensation to market conditions at least to some limited extent. We illustrate that while it is not optimal to tie worker compensation perfectly to market conditions, significant tying remains optimal.

We then focus on how the incentives to cheat on the cartel are affected if, to do so, a firm must hire more workers. A powerful and striking result obtains: For any cartel size, there are always market conditions for which firms can increase expected profits simply by *giving* workers a share of the profits that the firm earns. That is, the increased ability to collude in the output market more than offsets the reduced profit share that the firm receives.

The bottom line is that stock options and other profit-sharing agreements can do more than provide workers incentives to work hard; they can help firms support collusively higher prices in the output market.

## 5 Appendix

**Proof of Theorem 1:** We first consider strategies in which after the initial period, firms use strategies that require zero profit sharing, but otherwise, support profit-maximizing payoffs. We show that even with these sub-optimal strategies, that wages should be tied to market conditions in period 1. In this appendix,  $\Pi^s(\theta, a)$  refers to the sustainable level of profits in period one given these strategies when demand is  $\theta$ , and  $a$  is chosen. Hence  $K = E[\Pi^s(\theta, 0)/n]$  is the per-firm expected continuation profit in the absence of profit sharing.

Let  $Q(\theta, a)$  denote the associated profit-maximizing period one quantity produced when the demand shock is  $\theta$ , i.e., the output level that supports  $\Pi^s(\theta, a)$ :

$$Q(\theta, a) = \begin{cases} (\theta/2) & \theta \leq \theta^*(a) \\ \left(\theta + \sqrt{\theta^2 - \theta^*(a)^2}\right)/2 & \theta > \theta^*(a) \end{cases}$$

where  $\theta^*(a)$  is an implicit function of  $K$ , solving

$$\Pi^m(\theta^*(a), a) = \left(\frac{n}{n-1}\right) \left(\frac{\beta}{1-\beta}\right) K.$$

The worker's incentive compatibility constraint is

$$\int_{\underline{\theta}}^{\bar{\theta}} (\omega(a) + a(\theta - E(\theta))) Q(\theta, a) F(d\theta) \geq \int_{\underline{\theta}}^{\bar{\theta}} \bar{\omega} Q(\theta, a) F(d\theta).$$

In equilibrium, this constraint holds as an equality as firms never choose  $\omega(a)$  higher than necessary.

We prove the theorem using a sequence of lemmas.

**Lemma 1:** At  $a = 0$ ,

$$\frac{\partial \omega(a)}{\partial a} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)}.$$

**Proof:** Recall, as  $\bar{\omega} = 0$ ,

$$\int_{\underline{\theta}}^{\bar{\theta}} (\omega(a) + a(\theta - E(\theta))) Q(\theta, a) F(d\theta) = 0. \quad (5)$$

This expression establishes that  $\omega(0) = 0$ . Differentiating each side of (5) with respect to  $a$  obtains:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left( \frac{\partial \omega(a)}{\partial a} + \theta - E(\theta) \right) Q(\theta, a) + \frac{\partial Q(\theta, a)}{\partial a} (\omega(a) + a(\theta - E(\theta))) F(d\theta) = 0.$$

When evaluated at  $a = 0$ , the above expression becomes:

$$\frac{\partial \omega(a)}{\partial a} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)}.$$

■

**Lemma 2:** At  $a = 0$ ,

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)] = \int_{\underline{\theta}}^{\theta^*(0)} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta).$$

**Proof:** Note that

$$E[\Pi^s(\theta, a)] = \int_{\underline{\theta}}^{\theta^*(a)} \left( \frac{\theta + a(E(\theta) - \theta) - \omega(a)}{2} \right)^2 F(d\theta) + \int_{\theta^*(a)}^{\bar{\theta}} \left( \frac{n}{n-1} \right) \left( \frac{\beta}{1-\beta} \right) KF(d\theta).$$

By Leibnitz' rule, evaluated at  $a = 0$ , we obtain

$$\begin{aligned} \frac{\partial E[\Pi^s(\theta, a)]}{\partial a} &= \int_{\underline{\theta}}^{\theta^*(a)} \frac{\partial}{\partial a} \left( \frac{\theta + a(E(\theta) - \theta) - \omega(a)}{2} \right)^2 F(d\theta) \\ &= \int_{\underline{\theta}}^{\theta^*(a)} (\theta + a(E(\theta) - \theta) - \omega(a)) \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}}{2} \right) F(d\theta). \end{aligned}$$

Evaluating this derivative at  $a = 0$  yields

$$\int_{\underline{\theta}}^{\theta^*(0)} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta).$$

■

The above lemma guarantees that  $\frac{\partial}{\partial a} E[\Pi^s(\theta, a)]|_{a=0} > 0$  if  $\theta^*(0) \leq E(\theta) - \frac{\partial \omega(a)}{\partial a}|_{a=0}$ .

The rest of the proof considers the case where  $\theta^*(0) > E(\theta) - \frac{\partial \omega(a)}{\partial a}|_{a=0}$ .

**Lemma 3:**

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) < \int_{\underline{\theta}}^{\theta^*(0)} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta).$$

**Proof:** We note that

$$E(\theta) - \frac{\partial \omega(a)}{\partial a}|_{a=0} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)} > 0.$$

Moreover, note that the expression

$$\theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0}}{2} \right)$$

is quadratic, taking zeroes at  $\theta = 0$  and  $\theta = E(\theta) - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0}$ . The expression is strictly negative to the right of  $E(\theta) - \frac{\partial \omega(a)}{\partial a} \Big|_{a=0}$ . Therefore, the conclusion follows. ■

The next lemma guarantees that the increase in the fixed wage component is less than it would be if collusion were sustainable in all states of demand.

**Lemma 4:**

$$\frac{\partial \omega(a)}{\partial a} \Big|_{a=0} = \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) Q(\theta, 0) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} Q(\theta, 0) F(d\theta)} \leq \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta) (\theta/2) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) F(d\theta)}.$$

**Proof:** Suppose the wage tying parameter is  $a' > 0$ . The right-hand side of the expression in the statement of the lemma is the change in the fixed component of the wage under perfect monopoly pricing. Denote this fixed component of the wage under perfect monopoly pricing by  $\omega_M(a')$ . The right-hand side of the inequality is  $\frac{\partial \omega_M(a)}{\partial a} \Big|_{a=0}$ . Note that  $\omega(0) = \omega_M(0) = 0$ . We will show that

$$\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) < 0.$$

The left-hand side of the inequality is the expected wage workers would receive were monopoly pricing used in all states, and workers were offered contracts with fixed wage component  $\omega(a')$ .

This will enable us to show  $\omega_M(a') > \omega(a')$ . As  $a'$  is arbitrary, this establishes that at  $a = 0$ ,

$$\frac{\partial \omega(a)}{\partial a} \leq \frac{\partial \omega_M(a)}{\partial a},$$

as required. The incentive compatibility constraint for workers is:

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta, a')) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) = 0.$$

Separating:

$$\begin{aligned} & \int_{\underline{\theta}}^{\theta^*(a')} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) + \int_{\theta^*(a')}^{\bar{\theta}} (\theta/2) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) \\ & + \int_{\theta^*(a')}^{\bar{\theta}} \left( \sqrt{\theta^2 - \theta^*(a')^2} / 2 \right) (a'(\theta - E(\theta)) + \omega(a')) F(d\theta) = 0. \end{aligned}$$

We claim that

$$\int_{\underline{\theta}}^{\theta^*(a')} (\theta/2) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) < 0.$$

If not, it is easy to see that

$$\int_{\theta^*(a')}^{\bar{\theta}} (\theta/2) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) \geq 0,$$

which in turn implies

$$\int_{\theta^*(a')}^{\bar{\theta}} \left( \sqrt{\theta^2 - \theta^*(a')^2} / 2 \right) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

This follows as the last expression clearly weights higher values of  $\theta$  at a larger ratio than the first expression through the use of  $\left( \sqrt{\theta^2 - \theta^*(a')^2} / 2 \right)$  as opposed to  $(\theta/2)$ . Summing the three inequalities obtains

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta)) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) > 0,$$

a contradiction. As

$$\int_{\underline{\theta}}^{\theta^*(a')} (\theta/2) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) < 0,$$

it must be that

$$\int_{\theta^*(a')}^{\bar{\theta}} (\theta/2) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) + \int_{\theta^*(a')}^{\bar{\theta}} \left( \sqrt{\theta^2 - \theta^*(a')^2} / 2 \right) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

However, again, this inequality implies that

$$\int_{\theta^*(a')}^{\bar{\theta}} \left( \sqrt{\theta^2 - \theta^*(a')^2} / 2 \right) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

The preceding inequality is exactly

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta) - \theta/2) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) > 0.$$

Separating, we see

$$\int_{\underline{\theta}}^{\bar{\theta}} (Q(\theta)) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta) > \int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) (a' (\theta - E(\theta)) + \omega(a')) F(d\theta),$$

where the left hand side of the inequality is zero. Hence, we have established that  $\omega_M(a') > \omega(a')$  for any  $a' > 0$ ; moreover, then

$$\omega_M(a') - \omega_M(0) > \omega(a') - \omega(0),$$

establishing that at  $a = 0$ ,

$$\frac{\partial \omega_M(a)}{\partial a} \geq \frac{\partial \omega(a)}{\partial a}.$$

■

Finally, our theorem demonstrates that firms can always achieve higher expected sustainable profits by tying wages to market conditions. We show this by establishing that at  $a = 0$ ,

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)] > 0.$$

**Proof:** Suppose false. Then

$$\frac{\partial}{\partial a} E[\Pi^s(\theta, a)]|_{a=0} \leq 0.$$

The preceding lemmas taken together then imply that

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta \left( \frac{E(\theta) - \theta - \frac{\partial \omega(a)}{\partial a}|_{a=0}}{2} \right) F(d\theta) < 0$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta \left( \frac{E(\theta) - \theta - \frac{\int_{\underline{\theta}}^{\bar{\theta}} (E(\theta) - \theta)(\theta/2) F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} (\theta/2) F(d\theta)}}{2} \right) F(d\theta) < 0.$$

Thus, we see that

$$\int_{\underline{\theta}}^{\bar{\theta}} -\theta^2 + \frac{\theta \int_{\underline{\theta}}^{\bar{\theta}} \theta^2 F(d\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta F(d\theta)} F(d\theta) < 0$$

and thus

$$-E[\theta^2] + E[\theta] \left( \frac{E[\theta^2]}{E[\theta]} \right) < 0.$$

However, this is an obvious contradiction, as the expression above is identically zero. ■

Thus, we have shown that in period one, it is never profit-maximizing to set  $a = 0$ . This analysis extends to future periods, as the problem for firms is time separable. Further, since period profits are increased by wage tying, it follows that continuation profits would be increased if there is



wage tying in each period. In turn, increased continuation profits supports (weakly) more collusive output levels for all demand realizations, further increasing the profitability of tying wages to market conditions. ■

**Proof of Theorem 2:** First consider  $\theta \in H$ , so that monopoly pricing is not sustainable. In this region, firms price to realize an aggregate profit of  $K(\omega)$  in each state  $\theta$ . For a given price  $P$ , aggregate profits are  $(\theta - P)(P - \omega(\theta)) \equiv K(\omega)$ . We can solve this expression implicitly for  $P = \frac{\theta + \omega(\theta) - \sqrt{(\omega(\theta) - \theta)^2 - 4K(\omega)}}{2}$ . The associated market demand is  $\frac{\theta - \omega(\theta) + \sqrt{(\omega(\theta) - \theta)^2 - 4K(\omega)}}{2}$ , which implies that the total industry wage bill in state  $\theta$  is

$$\omega(\theta) \left[ \frac{\theta - \omega(\theta) + \sqrt{(\omega(\theta) - \theta)^2 - 4K(\omega)}}{2} \right]. \quad (6)$$

For  $\theta \in H$ , profit is limited by the incentive to deviate and does not depend on the wage. It follows that for a profit-maximizing wage profile, firms should maximize the expected wage to workers conditional on  $H$ . That is, the optimal wage profile maximizes the aggregate wage bill in each  $\theta \in H$ . Maximizing (6) with respect to  $\omega(\theta)$  yields the following first-order condition,

$$\left( \theta - \omega(\theta) + \sqrt{(\omega(\theta) - \theta)^2 - 4K} \right) + \omega(\theta) \left( -1 + \frac{-(\theta - \omega(\theta))}{\sqrt{(\omega(\theta) - \theta)^2 - 4K}} \right) = 0.$$

Multiplying each side of the first-order condition expression by  $\sqrt{(\omega(\theta) - \theta)^2 - 4K}$  yields an implicit solution for the optimal wage for  $\theta \in H$ , given the optimal sustainable monopoly profit, equation (4). Intuitively, to maximize workers' aggregate wage, firms trade off a decrease in the quantity demanded against the per-unit wage increase given that prices must generate exactly the maximal sustainable profit,  $K(\omega)$ .

Now consider wages set in states  $\theta \in L$ . Because firms extract all surplus, a worker's expected wage is zero (given the normalization). Hence, the expected wage bill in  $L$  must exactly offset the expected wage bill in  $H$ . For  $\theta \in L$ , firms can support the unconstrained monopoly profit. It follows that for  $\omega(\cdot)$  maximizes expected monopoly profit in the low region subject to the constraints that the aggregate expected wage is zero, and that wages are monotonically increasing.

$$\begin{aligned} & \max_{\{\omega(\theta): \theta \in L\}} \int_L \left( \frac{\theta - \omega(\theta)}{2} \right)^2 F(d\theta) \\ \text{s.t.} \quad & \int_H \left( \frac{\theta - \omega(\theta) + \sqrt{(\omega(\theta) - \theta)^2 - 4K(\omega)}}{2} \right) \omega(\theta) F(d\theta) + \int_L \omega(\theta) \left( \frac{\theta - \omega(\theta)}{2} \right) F(d\theta) = 0 \\ & \omega(\theta) \geq \omega(\tilde{\theta}), \quad \theta \geq \tilde{\theta}. \end{aligned}$$

Suppose the monotonicity constraint does not bind. Then setting up a Lagrangean and differentiating with respect to  $\omega(\theta)$  state-by-state yields

$$2(\theta - \omega(\theta)) + \lambda(\theta - 2\omega(\theta)) = 0.$$

Solving,

$$\omega(\theta) = \theta \frac{(\lambda - 2)}{2(\lambda - 1)} < \theta.$$

That is, the first constraint implies  $\int_L \omega(\theta) F(d\theta) < 0$ , which implies that  $\frac{\lambda-2}{2(\lambda-1)} < 0$ . But, this means that on any interval  $\omega(\theta)$  is declining in  $\theta$ , violating monotonicity. It follows that the optimal wage in  $L$  is a constant. The remainder of the theorem follows straightforwardly. ■

**Proof of Theorem 3:** We show that under the conditions detailed in the theorem that setting  $a > 0$  is profit-maximizing even when (i) firms produce the monopoly level of output, where-ever possible in the first period, and (ii) as in the proof of the theorem 1, there is a reversion after the first period to the strategies that do not feature profit sharing, and correspond to those in Rotemberg and Saloner. Fixing continuation payoffs to correspond to the Rotemberg and Saloner monopoly pricing equilibrium, for any  $a$ , we may solve for  $\theta^*(a)$ , as the  $\theta^*$  which solves

$$(1 - anL(\theta^*, a)) \Pi^m(\theta^*, a) = \frac{(1 - aL(\theta^*, a)) \Pi^m(\theta^*)}{n} + K,$$

where  $K = E[\Pi^s(\theta)/n]$  is the expected period profit for any given firm. To simplify notation, we write  $\theta^*$ , leaving the dependence on  $a$  implicit. Differentiating both sides of the above equality with respect to  $a$  and evaluating at  $a = 0$  obtains:

$$\frac{\partial \Pi^m(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a} - n \Pi^m(\theta^*) L(\theta^*, a) = \frac{-L(\theta^*, a) \Pi^m(\theta^*) + \frac{\partial \Pi^m(\theta^*, a)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a}}{n}.$$

Rearranging yields

$$\frac{\partial \Pi^m(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a} = (n + 1) \Pi^m(\theta^*) L(\theta^*, a).$$

Recall that expected period profits for the cartel given profit sharing level  $a$  are

$$\int_{\underline{\theta}}^{\theta^*} (1 - aL(\theta, a)) \Pi^m(\theta) F(d\theta) + \int_{\theta^*}^{\bar{\theta}} (1 - aL(\theta, a)) \Pi^m(\theta^*) F(d\theta).$$

Differentiating this expression with respect to  $a$ , and evaluating at  $a = 0$  yields:

$$\int_{\underline{\theta}}^{\theta^*} -L(\theta, a) \Pi^m(\theta) F(d\theta) + \int_{\theta^*}^{\bar{\theta}} -L(\theta, a) \Pi^m(\theta^*) + \frac{\partial \Pi^m(\theta^*)}{\partial \theta^*} \frac{\partial \theta^*}{\partial a} F(d\theta).$$

We establish when this function takes positive values, or (solving) when

$$\int_{\underline{\theta}}^{\theta^*} -L(\theta, a) \Pi^m(\theta) F(d\theta) + \int_{\theta^*}^{\bar{\theta}} \Pi^m(\theta^*) [(n+1)L(\theta^*, a) - L(\theta, a)] F(d\theta)$$

is strictly positive. Substituting for  $L(\theta, a)$  and  $\Pi^m(\theta)$ , as derived in the demand-linked wage contracts case and normalizing, yields the following expression:

$$\frac{\theta^4 - \theta^{*4}}{4} + (1/2) \theta^{*2} \left[ 2(n+1) \bar{\theta} \theta^* - (2n+1) \theta^{*2} - \bar{\theta}^2 - \bar{\theta} \sqrt{\bar{\theta}^2 - \theta^{*2}} + \theta^{*2} \log \left( \frac{\bar{\theta} + \sqrt{\bar{\theta}^2 - \theta^{*2}}}{\theta^*} \right) \right].$$

Rearranging, we see that this expression is positive if:

$$\left[ 2(n+1) \bar{\theta} \theta^* - (2n+1) \theta^{*2} - \bar{\theta}^2 - \bar{\theta} \sqrt{\bar{\theta}^2 - \theta^{*2}} + \theta^{*2} \log \left( \frac{\bar{\theta} + \sqrt{\bar{\theta}^2 - \theta^{*2}}}{\theta^*} \right) \right] > \frac{\theta^{*4} - \theta^4}{2\theta^{*2}}. \quad (7)$$

When this expression is positive, expected period one profits can be raised by signing profit sharing contracts with workers, and using a monopoly pricing scheme. We see that continuation payoffs must also be driven up, as the analysis applies at all periods. Therefore, we conclude that all firms setting  $a = 0$  is not profit-maximizing.

1. Note that the derivative of (7) with respect to  $\bar{\theta}$  evaluated at  $\bar{\theta}$  when  $\theta^* = \underline{\theta}$  is equal to  $2(n+1)\theta^* - 2\theta - 2(\theta^2 - \theta^{*2})^{1/2}$ . Therefore, there exists  $n$  large enough so that this expression is positive, for all  $\theta \leq \bar{\theta}$ . For such  $n$ , there thus exists a neighborhood of  $\underline{\theta}$  such that for all  $\theta$  in this neighborhood, (7) is satisfied. Let  $n(\underline{\theta}, \bar{\theta}, \beta)$  be this  $n$ . As  $n$  increases, we see that (7) is true for any  $\theta^*$  for which it was previously true (a simple monotonicity argument establishes this). Moreover, as  $n$  increases,  $\theta^*$  decreases to zero. Hence, the result holds.
2. This follows since at  $\theta^*(0) = \bar{\theta} = \underline{\theta}$ , both sides of (7) are equal to zero, but differentiating the left-hand side with respect to  $\bar{\theta}$  yields  $(2n)\bar{\theta} > 0$ , while the derivative of the right-hand side is zero.
3. This result is a corollary to (2), as  $\theta^*$  is an increasing function of  $\beta$ . ■

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