Inequality aversion and risk aversion

Christopher P. Chambers

Conference on Inequality and Risk, Paris, June 26, 2010

Introduction

- Risk attitudes for groups or households
- Samuelson's social welfare justification of representative consumer hypothesis
- A society allocating resources to maximize social welfare behaves as a single agent
- Optimal allocation of aggregate risk in a household

- Samuelsonian aggregation in this context: risk sharing/risk allocation
- Social welfare functions can be compared with respect to inequality aversion
- Is there a general relationship between inequality aversion and household risk aversion?

- To talk about inequality aversion, need interpersonally comparable notion of utility
- In the risk case, a natural benchmark is the *certainty equivalent*
- Certainty equivalent is calibrated so that utility of a riskless prospect is the value of that prospect

- Simple intuition suggests more inequality averse social welfare functions imply more risk averse households
- Intuition is confirmed in some basic cases
- In comparing "absolutely inequality averse" social welfare functions
- But the intuition does not extend more generally
- In fact, *inequality neutrality* generates the least risk averse households of all

Basic setup

- $\Omega = \{1, ..., \omega\}$ finite set of states of the world
- $N = \{1, ..., n\}$ finite set of agents
- Consumption space is $\mathbb{R}^{\Omega}_{+}$
- Prior $\pi = (\pi_1, ..., \pi_\omega)$ over states of the world (say full support)

- Preferences R on $\mathbb{R}_+^\Omega$ are increasing and continuous; they are also *risk averse*
- Risk aversion requires that for all $x \in \mathbb{R}_+^\Omega$, $(\pi \cdot x)\, R\, x$
- Every preference can be represented by its *certainty equivalent function U^i*
- $U^i : \mathbb{R}_+^\Omega \to \mathbb{R}_+$ satisfies $\left(U^i(x), ..., U^i(x)\right) I\, x$

- Fixed-profile aggregation exercise
- To keep analysis simple, assume $W : \mathbb{R}^N_+ \rightarrow \mathbb{R}$ operates directly on utils (in terms of certainty equivalents)
- Thus group utility of allocation $(x^1, ..., x^n)$ is $W(U^1(x^1), ..., U^n(x^n))$

- Household utility becomes $U^W : \mathbb{R}_+^\Omega \to \mathbb{R}$ defined by

$$U^W(x) = \max_{\sum_i x^i = x} W(U^1(x^1), \ldots, U^n(x^n))$$

- Household maximizes social utility across all allocations of x
- Note: U^W not necessarily a certainty equivalent representation

A basic example

Let's consider three social welfare functions:

A basic example

Let's consider three social welfare functions:

1. Maxmin: $W_{min}(u^1, ..., u^n) = \min_i u^i$

A basic example

Let's consider three social welfare functions:

1. Maxmin: $W_{min}(u^1, ..., u^n) = \min_i u^i$
2. Utilitarian: $W_U(u^1, ..., u^n) = \sum_i u^i$

A basic example

Let's consider three social welfare functions:

1. Maxmin: $W_{min}(u^1, ..., u^n) = \min_i u^i$
2. Utilitarian: $W_U(u^1, ..., u^n) = \sum_i u^i$
3. Maxmax: $W_{max}(u^1, ..., u^n) = \max_i u^i$

- Let's assume for all i, U^i is homogeneous: $U^i(\alpha x) = \alpha U^i(x)$ for $\alpha > 0$ and quasiconcave
- Fix a riskless aggregate bundle $(c, c, ..., c) \in \mathbb{R}^\Omega_+$
- Can compare risk aversion of different induced household preferences by studying at least as good as sets for $(c, c, ..., c)$.

- For each agent i, denote by $\overline{U}^i(c) = \{x \in \mathbb{R}^\Omega_+ : U^i(x) \geq U^i(c, c, ..., c)\}$
- The at least as good as set

Now compute at least as good as set for U^W at $(c, c, \ldots, c)$ for each of the three social welfare functions

1. Maxmin: $\sum_i (1/n)\overline{U}^i(c)$

Now compute at least as good as set for U^W at $(c, c, \ldots, c)$ for each of the three social welfare functions

1. Maxmin: $\sum_i (1/n)\overline{U}^i(c)$
2. Utilitarian: $conv \bigcup_i \overline{U}^i(c)$

Now compute at least as good as set for U^W at $(c, c, \ldots, c)$ for each of the three social welfare functions

1. Maxmin: $\sum_i (1/n)\overline{U}^i(c)$
2. Utilitarian: $conv \bigcup_i \overline{U}^i(c)$
3. Maxmax: $\bigcup_i \overline{U}^i(c)$

Now compute at least as good as set for U^W at $(c, c, ..., c)$ for each of the three social welfare functions

1. Maxmin: $\sum_i (1/n)\overline{U}^i(c)$
2. Utilitarian: $conv \bigcup_i \overline{U}^i(c)$
3. Maxmax: $\bigcup_i \overline{U}^i(c)$

- Maxmin, Maxmax, both subset of utilitarian
- No relation between maxmin and maxmax (in general)
- The "inequality neutral" social welfare function results in least risk averse household preference

A few formal definitions

- Standard Arrow-Pratt comparative notions of inequality aversion and risk aversion
- Utility U more risk averse than U' if for all $(c, c, ..., c) \in \mathbb{R}_+^\Omega$

$$U(x) \geq U(c, ..., c) \Rightarrow U'(x) \geq U'(c, c, ..., c)$$

- Social welfare W more inequality averse than W' if for all $(u, u, ..., u) \in \mathbb{R}_+^N$

$$W(y) \geq W(u, ..., u) \Rightarrow W'(y) \geq W'(u, ..., u)$$

- Definitions appear similar, but relate to different spaces ($\mathbb{R}_+^\Omega$ vs. $\mathbb{R}_+^N$)

- Further, social welfare function W is *inequality averse* (in an absolute sense) if for all $(u^1, ..., u^n)$

$$W(\sum_i \frac{1}{n} u^i, ..., \sum_i \frac{1}{n} u^i) \geq W(u^1, ..., u^n)$$

- In environments of certainty, we should always allocate equitably (follows from certainty equivalent representation)
- Dividing a (certain) dollar equally is always at least as good as any other allocation

Theorem

If W and W' are inequality averse, and W is more inequality averse than W', then for any $(U^1, ..., U^n)$, U^W is more risk averse than $U^{W'}$.

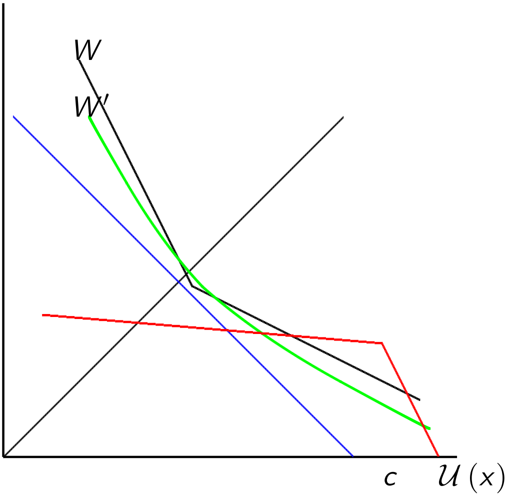

Figure: Proof of main result

- Does result extend to *non* inequality averse social welfare functions?
- No (by the example in the beginning)
- But we do have one result

Theorem

For any social welfare function W, the household preference induced by the utilitarian welfare function W_U is less risk averse than that induced by W.

Conclusion

- We conclude that inequality neutral social welfare functions induce least risk averse societies/households
- An inequality neutral social welfare function is a function of total amount of money held by society (in riskless environments)
- Thus, a society which tends to maximize national income, etc, will tend to be as risk neutral as possible (it will be risk averse whenever individuals are all risk averse)