Preference aggregation with incomplete information

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September 20, 2010

Abstract

We show that in an environment of incomplete information, if individual preferences are sufficiently rich, then monotonicity and the Pareto property applied only when there is common knowledge of Pareto dominance imply (i) there must exist a common prior over the smallest common knowledge event and (ii) aggregation must be ex-ante utilitarian with respect to that common prior. This work builds on the contributions of Nehring [12], de Clippel [4], and Zuber [16].

1 Introduction

This note addresses preference aggregation under incomplete information. We imagine a collection of agents, each of whom faces uncertainty both as to an objective state of the world, and as to the preferences of the other agents in society. The agents' preferences in the absence of uncertainty are commonly known. Thus, we have an environment of interactive uncertainty. We use the standard Harsanyi [8] notion of a type space. Each agent is endowed with a partition over the type space; and each type is identified with a probability measure over the element of the partition containing the type, together with the objective states.

We view a type space as a method of representing interacting hierarchies of beliefs. That is, any type space induces a hierarchy of beliefs about the preferences of all agents, and conversely, any hierarchy of beliefs can be defined as a type in some type space.[1] Thus, many environments of interactive uncertainty can be represented by this simple device. We imagine that there is one "true" type and the type space is merely a representation of the hierarchy of beliefs specified by the interactive uncertainty.

Agents have preferences over acts. An act is any outcome which depends on the true state and preferences of other agents. To this end, we want to understand how *group* preferences should be constructed. In a seminal work, Nehring [12] identifies two very weak criteria. His first criterion requires that if it is commonly known that act f Pareto dominates act g, then f should be ranked socially at least as good as g. Note that the hypothesis that it is commonly known that f Pareto dominates g is much weaker than the hypothesis that f Pareto dominates g. In particular, it requires not only that f Pareto dominate g, but that everybody knows this to be true, everybody knows that everybody knows this to be true, and so forth. Nehring suggests that a group of agents deciding on a collective preference can only use information which is commonly known in constructing this preference.

Nehring's second criterion is a basic monotonicity property, akin to P3 of Savage [14], or monotonicity of Anscombe and Aumann [2]. The monotonicity property allows one to meaningfully discuss a "state-independent" ranking of certain payoffs. Nehring shows that, under the Pareto assumption, and when this state-independent ranking is utilitarian (with respect to individual von Neumann Morgenstern utilities), then there are two immediate implications. First, society *must* possess a common prior.[2] Secondly, the social ranking must be *ex-ante* utilitarian with respect to that common prior.

Ex-ante utilitarianism refers to a *hypothetical* ex-ante situation in which types are

[1] On this, see Mertens and Zamir [10] and Brandenburger and Dekel [3].

[2] Here, a common prior is any probability measure over the type space cross state space which induces the system of beliefs as conditional probabilities.

unknown, and a common prior is given over those types. Obviously, as a type space is simply a representation of preference, this ex-ante situation is never actually faced by the agents. Nevertheless, the language referring to ex-ante situations and ex-interim situations is commonly used and well understood. We adopt this terminology here with the understanding that type spaces merely represent preference.

There are two other direct predecessors to this note. de Clippel [4] builds on Nehring [12], establishing that, in many compelling cases, when the state-independent ranking is *not* utilitarian, impossibility results obtain, in many cases *even when* there is a common prior. In particular, he establishes that, for a broad class of common priors, social welfare functions cannot be based on an ex-post criterion satisfying the Pigou-Dalton transfer property.

Zuber [16] works in a standard setup where each agent knows the preferences of the remaining agents, but there is uncertainty as to the true state of the world. He had the insight that monotonicity of social preference and the ex-ante Pareto property jointly imply a *separability* property. That is, social preference can be determined by first finding the social utility within each state, and then aggregating across states. Alternatively, since the ex-ante Pareto property is satisfied, we can find social utility by first finding each individual ex-ante utility, and then aggregating. It is well-known, at least since Gorman [7] that separability in two dimensions in this sense implies a form of additive separability. Thus, he derives the *implication* (which is *assumed* here) that each agent must have preferences which are additively separable across states, and the social welfare ranking must be additively separable across agents. In short, he shows that the social ranking *must* be utilitarian.

Now, Nehring [12]'s version of the Pareto principle is much weaker (in general) than the ex-ante Pareto property. However; there is still a type of separability present. That is, if we can compute the so-called *interim* expected utilities for each agent and each cell of each agent's partition, we can compute social welfare. Our contribution here is to show that results of Gorman [7] and Von Stengel [15] can be applied in this environment to show that additive separability of the social welfare function can still be obtained on the smallest common knowledge event containing the true type, so long as there are two objective states. Hence, we again arrive at a form of utilitarianism for social preference.

As a consequence of Nehring [12], we therefore establish that there exists a *common prior* (over the common knowledge event) and that the social welfare function is a form of *ex-ante utilitarianism* with respect to this common prior. That is, the social welfare function computes expected utilities under the common prior, and then adds these. We emphasize that, in our work, there is no meaningful *ex-ante* stage where the common prior has an operational meaning in terms of willingness to bet. Instead, it is known that the existence of a common prior is equivalent to a statement about the possibility of speculative trade, as in Morris [11].

Section 2 introduces the model and main result. Section 3 concludes.

2 The model

N denotes a finite set of *agents*, where $|N| \geq 2$. Given is a finite set of *types*, $\mathcal{T}$, and a finite set of *states*, Θ, where $|\Theta| \geq 2$.[3] A type is a method of representing hierarchies of beliefs about Θ; and as such is a description of a preference profile for interactive uncertainty. Each agent is endowed with a partition Π_i of $\mathcal{T}$. We assume that $\bigwedge \Pi_i = \{\mathcal{T}, \varnothing\}$; that is, $\mathcal{T}$ is the smallest common knowledge event. This fact will be important for our later result.

Given is a set of *outcomes* X. An *act* is a mapping $f : T \times \Theta \to X$. The set of acts is denoted $\mathcal{F}$. Thus, the payoff of an act is public, and can depend not only on the state of the world, but the preferences of the agents.

Each agent is endowed with a preference relation $\succeq_i^t$, which is a mapping carrying

[3]There is real content imposed on the underlying hierarchy of beliefs by assuming a finite type space. However, in at least one natural topology on the space of hierarchies of beliefs, finite type spaces are dense Mertens and Zamir [10], Theorem 3.1.

each agent and type to a complete, transitive binary relation over $\mathcal{F}$.

We assume that there exists, for each agent, a mapping $p_i : \pi_i \mapsto \Delta(\pi_i \times \Theta)$, carrying each element of i's partition to a full support probability measure over that element of the partition and set of states of the world. For $\pi_i \in \Pi_i$, we denote this probability measure by $p_i(\cdot|\pi_i)$. We also assume there exists a function $u_i : X \to \mathbb{R}$ such that for all $f \in \mathcal{F}$, the function

$$U_i^t(f) = E_{p_i}[u_i \circ f|\pi_i(t)] = \sum_{t' \in \pi_i(t)} \sum_{\theta \in \Theta} p_i(t', \theta|\pi_i(t)) u_i(f(t', \theta)).$$

represents $\succeq_i^t$, where $\pi_i(t)$ is the element of the partition Π_i containing t. Note therefore that if $t, t' \in \pi_i$, then $\succeq_i^t = \succeq_i^{t'}$.

We further assume that for all $y \in \mathbb{R}^N$, there exists $x \in X$ for which $u_i(x) = y_i$. This richness condition can be weakened, but not too much.

Although it is not necessary, no substantive results are lost by assuming that $\succeq_0$ is commonly known (that is, does not depend on t). In our environment, it seems reasonable to allow for the possibility that agents in society do not know the social ranking. All of our results extend naturally to this case, so long as each of the following axioms are required to hold for all $t \in \mathcal{T}$.

Nehring suggests the following two properties, which are of interest even in this general framework.

The first is the following variant of the Pareto property, which we generalize to our environment:

Common Pareto: If the event $\{t \in \mathcal{T} : f \succeq_i^t g \text{ for all } i\}$ is commonly known, then $f \succeq_0 g$. If the event $\{t \in \mathcal{T} : f \succeq_i^t g \text{ for all } i\}$ is commonly known, and $\{t \in \mathcal{T} : g \succeq_i^t f \text{ for all } i\}$ is not commonly known, then $f \succ_0 g$.[4]

Common Pareto is the requirement that, if it is common knowledge that f is better than g for everybody, it should be common knowledge that society ranks f

[4]The hypothesis of this axiom, that it is commonly known that f is better than g for all $i \in N$, is essentially the notion of interim dominance discussed in Holmström and Myerson [9].

better than g. If, in addition, it is not commonly known that g is better than f, it requires society should rank f strictly above g. Nehring [12] justifies this axiom on the grounds that a group of individuals constructing a social preference should only be able to use information which is commonly available. The second part of the axiom as stated here is not exactly as it appears in his work, but this form of the axiom is also compelling (that is, if it is not commonly known that g is better than f, then someone thinks it's possible that f is strictly better than g for some agent (perhaps after several more iterations of possibility).

Monotonicity: $\succeq_0$ satisfies monotonicity: that is, if $f(t,\theta) \succeq_0 g(t,\theta)$ for all $(t,\theta) \in \mathcal{T} \times \Theta$, then $f \succeq_0 g$.

We will also need some form of continuity.

Continuity: For all $f \in \mathcal{F}$, $\{(u_i(g(t,\theta)))_{(i,t,\theta)} \in \mathbb{R}^{N\times\mathcal{T}\times\Theta} : g \succeq_0 f\}$ and $\{(u_i(g(t,\theta)))_{(i,t,\theta)} \in \mathbb{R}^{N\times\mathcal{T}\times\Theta} : f \succeq_0 g\}$ are closed in the standard Euclidean topology.

Now, we say that $p \in \Delta(\mathcal{T} \times \Theta)$ is a *common prior* if for all $i \in N$ and all $\pi_i \in \Pi_i$,

$$p_i(\cdot|\pi_i) = p(\cdot|\{t \in \pi_i \text{ and } \theta \in \Theta\}).$$

A common prior here has no meaning in any kind of "ex-ante" stage over types, as there is no ex-ante stage over types. But mathematically, it is convenient.

Our main result is that if common Pareto and monotonicity are satisfied, then aggregation must be utilitarian. As a consequence, there must exist a common prior and aggregation must be *ex-ante* utilitarian. That is, there exists a prior $p \in \Delta(\mathcal{T}\times\Theta)$ and for all $i \in N$, $\lambda_i > 0$ for which

$$f \succeq_0 g \Leftrightarrow \sum_{i\in N} \lambda_i \sum_{t\in\mathcal{T}} \sum_{\theta\in\Theta} p(t,\theta) u_i(f(t,\theta)) \geq \sum_{i\in N} \lambda_i \sum_{t\in\mathcal{T}} \sum_{\theta\in\Theta} p(t,\theta) u_i(g(t,\theta)).$$

In what follows, $\sum_{t\in\mathcal{T}} \sum_{\theta\in\Theta} p(t,\theta)u_i(f(t,\theta))$ is abbreviated $E_p[u_i \circ f]$. The result is therefore a generalization of Zuber [16]: monotonicity of social preference in situations of uncertainty is highly restrictive.

Theorem 1. *Common Pareto, monotonicity, and continuity are satisfied if and only if there is a common prior p and for each $i \in N$, there exists $\lambda_i > 0$ such that $f \succeq_0 g$ if and only if $\sum_{i \in N} \lambda_i E_p[u_i \circ f] \geq \sum_{i \in N} \lambda_i E_p[u_i \circ g]$.*

Mathematically, the contribution here over Zuber [16] is to weaken the Pareto property to common Pareto. The insight that monotonicity for social preference is a form of separability is due to him.

Proof. First, note that by common Pareto, for any pair $f, g \in \mathcal{F}$, if for all $i \in N$ and all (t, θ), $u_i(f(t, \theta)) = u_i(g(t, \theta))$, it follows that $f \sim_0 g$. Hence, the induced binary relation R on $\mathbb{R}^{N \times \mathcal{T} \times \Theta}$ defined by $x \, R \, y$ if and only if there exists $f, g \in \mathcal{F}$ for which $f \succeq_0 g$, $u_i(f(t, \theta)) = x_{(i,t,\theta)}$ and $u_i(g(t, \theta)) = y_{(i,t,\theta)}$ is well-defined. By continuity, it is a continuous binary relation. Because $\mathbb{R}^{N \times \mathcal{T} \times \Theta}$ is connected and separable, we know by Debreu [6] that there exists a continuous function $V : \mathbb{R}^{N \times \mathcal{T} \times \Theta} \to \mathbb{R}$ representing R, and that further

$$U_0(f) = V\left((u_i(f(t, \theta)))_{(i,t,\theta) \in N \times \mathcal{T} \times \Theta}\right),$$

represents $\succeq_0$. It should be noted that by common Pareto, V is strictly monotonic in all components.

For $M \subseteq N \times \mathcal{T} \times \Theta$, say that V is *separable with respect to* M if there exist functions $W : \mathbb{R} \times \mathbb{R}^{(N \times \mathcal{T} \times \Theta) \setminus M} \to \mathbb{R}$ and $h : \mathbb{R}^M \to \mathbb{R}$ for which $V(x_M, x_{-M}) = W(h(x_M), x_{-M})$. Our goal is to show that V is strictly monotonic in all components and separable with respect to every two-element subset of $N \times \mathcal{T} \times \Theta$. This will allow us to invoke Debreu's theorem [5].

First, note that for any $i \in N$, $t \in \mathcal{T}$ and any $\theta, \theta' \in \Theta$, V is separable with respect to $\{(i, t, \theta), (i, t, \theta')\}$. This follows by common Pareto: by common Pareto, if for all $i \in N$ and all $\pi_i \in \Pi_i$, $E_{p_i}[u_i \circ f | \pi_i] = E_{p_i}[u_i \circ g | \pi_i]$, it follows that $f \sim_0 g$. Consequently, by setting $h : \mathbb{R}^{\{(i,t,\theta),(i,t,\theta')\}} \to \mathbb{R}$ as

$$h(x, y) = p_i(t, \theta | \pi_i) x + p_i(t, \theta' | \pi_i) y,$$

we see that separability holds for this pair.

By a similar argument, for any $i \in N$, $\theta \in \Theta$ and $t' \in \pi_i(t)$, separability holds for the set $\{(i,t,\theta),(i,t',\theta)\}$.

Finally, for any pair $i,j \in N$ and any $\theta \in \Theta$ and $t \in \mathcal{T}$, separability holds for the set $\{(i,t,\theta),(j,t,\theta)\}$ for each $i \in N$. This will follow as a consequence of monotonicity and the preceding two results. We first claim that for any $t \in \mathcal{T}$, $\theta \in \Theta$, separability holds with respect to the set $\{(i,t,\theta) : i \in N\}$. To see why, note that for any constant acts $x, y \in \mathcal{F}$, Pareto (in the traditional sense) is satisfied. Consequently, for such acts, there exists a function $W : \mathbb{R}^N \to \mathbb{R}$ such that $x \succeq_0 y$ if and only if $W(u_1(x),...,u_n(x)) \geq W(u_1(y),...,u_n(y))$. Therefore, by monotonicity, $V(x) = T\left((W(x(t,\theta))_{(t,\theta)\in\mathcal{T}\times\Theta})\right)$ for some function $T : \mathbb{R}^\Theta \to \mathbb{R}$, where $x(t,\theta) = (x(i,t,\theta))_{i\in N}$. This in fact establishes separability with respect to $\{(i,t,\theta) : i \in N\}$.

We now apply Theorem 12 and Proposition 16 of Von Stengel [15][5], which establish that for any given function V, the family of index sets with respect to which V is separable is closed under (1) overlapping union and (2) intersection.

So fix $j,k \in N$ and $(t,\theta) \in \mathcal{T} \times \Theta$. Let $\theta' \in \Theta$, $\theta' \neq \theta$. By the preceding results and [15] closure result, V is separable with respect to

$$L = \{(j,t,\theta),(j,t,\theta')\} \cup \{(i,t,\theta') : i \in N\} \cup \{(k,t,\theta),(k,t,\theta')\}$$

as well as $\{(i,t,\theta) : i \in N\}$; now $\{(i,t,\theta) : i \in N\} \cap L = \{(j,t,\theta),(k,t,\theta)\}$. Hence V is separable with respect to $\{(j,t,\theta),(k,t,\theta)\}$.

Lastly, we must show that for *any* $t,t' \in \mathcal{T}$, V is separable with respect to $\{(i,t,\theta),(i,t',\theta)\}$. Note that we have already shown that this is true when $t' \in \pi_i(t)$.

This is where the assumption that $\mathcal{T}$ is common knowledge is required. A finite sequence $\{(i_m,t_m)\}_{m=1}^{M} \subseteq N \times \mathcal{T}$ for which $(i_1,t_1) = (i,t)$ and $(i_m,t_m) = (i,t')$ and for all $m \in \{1,...,M-1\}$, V is separable with respect to $\{(i_m,t_m),(i_{m+1},t_{m+1})\}$ is termed a *separable path* between (i,t) and (i,t'). But the definition of common

[5] These type of separability results have a long history, dating back to [7].

knowledge implies the statement that for any $t, t' \in \mathcal{T}$, there exists a separable path between (i,t) and (i,t').[6] Now, let $\{(i_m, t_m)\}_{m=1}^{M}$ be a separable path between (i,t) and (i,t'). By [15], V is separable with respect to $P = \bigcup_i \{(i_m, t_m, \theta)\}_{m=1}^{M}$. Let $\theta' \in \Theta$, where $\theta' \neq \theta$. Then V is also separable with respect to $R = \{(i,t,\theta),(i,t,\theta')\} \cup \{(i,t',\theta'),(i,t',\theta)\} \cup \bigcup_i \{(i_m, t_m, \theta')\}_{m=1}^{M}$. Therefore, V is separable with respect to $R \cap P = \{(i,t,\theta),(i,t',\theta)\}$.

Again by [15], this allows us to conclude that V is separable with respect to $\{(i,t,\theta) : (t,\theta) \in \mathcal{T} \times \Theta\}$. And from here it is easy to conclude that V is separable with respect to any set (equivalently any two-element set). First, we show that V is separable with respect to any pair $\{(i,t,\theta),(i,t',\theta')\}$. Fix $j \neq i$. This follows as V is separable with respect to

$$\{(k,t,\theta)|k \in N\} \cup \{(j,t'',\theta'') : (t'',\theta'') \in \mathcal{T} \times \Theta\} \cup \{(k,t',\theta')|k \in N\}$$

and

$$\{(i,t'',\theta'') : (t'',\theta'') \in \mathcal{T} \times \Theta\},$$

and the intersection of these two sets is precisely $\{(i,t,\theta),(i,t',\theta')\}$.

Now fix any $\{(i,t,\theta),(j,t',\theta')\}$. Note that V is separable with respect to $\{(i,t,\theta),(j,t,\theta),(j,t',\theta')\}$ and $\{(i,t,\theta),(i,t',\theta'),(j,t',\theta')\}$; as well as the intersection of these two sets, which is $\{(i,t,\theta),(j,t',\theta')\}$. So V is therefore separable for all subsets of $N \times \mathcal{T} \times \Theta$.

By Debreu [5], we establish that V is ordinally equivalent to an additively separable function $\tilde{V}$; that is, $\tilde{V}(x) = \sum_{(i,t,\theta) \in N \times \mathcal{T} \times \Theta} \varphi_{(i,t,\theta)}(x_{(i,t,\theta)})$, for some continuous

[6] To see why, note that each Π_i on $\mathcal{T}$ is equivalently defined by an equivalence relation E_i, whereby $t\ E_i\ t'$ if and only if $t' \in \pi_i(t)$. The partition $\bigwedge \Pi_i$ is the finest common coarsening of the partitions $\{\Pi_i\}_{i \in N}$; equivalently, the smallest equivalence relation E for which for all i, $E_i \subseteq E$. Since $\bigcup_i E_i$ is symmetric and reflexive, E must be the transitive closure of $\bigcup_i E_i$, which means that there exists $t_1, ..., t_K$ and $i_1, ..., i_{K-1}$ such that $t = t_1\ E_{i_1}\ t_2\ E_{i_2}\ t_3 ... t_{K-1}\ E_{i_{K-1}}\ t_K = t'$. Then $\{(i,t_1),(i_1,t_1)\}, \{(i_1,t_1),(i_1,t_2)\}, \{(i_1,t_2),(i_2,t_2)\}, \{(i_2,t_2),(i_2,t_3)\}, ..., \{(i_K,t_K),(i,t_K)\}$ is a separable path.

functions $\varphi_{(i,t,\theta)}$, where $\tilde{V} = \psi \circ V$ for some strictly increasing ψ.

Step 2: Establishing that each φ is affine

This follows from a standard Pexider equation argument. Note that, for each $i \in N$ and each $\pi_i \in \Pi_i$,

$$E_{p_i}[x|\pi_i] = \sum_{t\in\pi_i}\sum_{\theta\in\Theta} p_i(t,\theta|\pi_i)x(i,t,\theta)$$

represents preference conditional on the event $\{(t,\theta) : t \in \pi_i\}$, as does

$$\sum_{\pi_i\times\Theta} \varphi_{(i,t,\theta)}(x_{(i,t,\theta)}).$$

So

$$\sum_{\pi_i\times\Theta} \varphi_{(i,t,\theta)}(x_{(i,t,\theta)}) = \psi\left(\sum_{t\in\pi_i}\sum_{\theta\in\Theta} p_i(t,\theta|\pi_i)x(i,t,\theta)\right) \tag{1}$$

for some ψ strictly increasing. A standard Pexider argument[7] therefore implies that

$$\varphi_{(i,t,\theta)}(x) = \alpha_{(i,\pi_i)}p_i(t,\theta|\pi_i)x + \beta_{(i,t,\theta)}.$$

As each $\beta_{(i,t,\theta)}$ is a constant term, we can discard them and end up with an ordinally equivalent ranking.

Consequently, $\tilde{V}$ on the domain of constant acts is ordinally equivalent to:

$$\tilde{V}(x) = \sum_{i\in N}\sum_{\pi_i}\sum_{t\in\pi_i,\theta\in\Theta} \alpha_{(\pi_i(t),t)}x_i.$$

In other words, letting $\alpha_i^* = \sum_{\pi_i}\sum_{t\in\pi_i,\theta\in\Theta}\alpha_{(\pi_i(t),t)}$, $\sum_{i\in N}\alpha_i^* u_i(x)$ represents social preference over constant acts, a utilitarian criterion. It obviously represents social preference within each state as well. We now can use a classic separation theorem

[7] Equation 1 is a Pexider equation after redefining $y(i,t,\theta) = p_i(t,\theta|\pi_i)x(i,t,\theta)$ and $\hat{\varphi}_{(i,t,\theta)}(y) = \varphi_{(i,t,\theta)}(y/p_i(t,\theta|\pi_i))$. As each φ is continuous, its solution is well-known, see for example Aczél [1], p. 42 Theorem 1.

(see [12] Theorem 2) to show that there exists a common prior.[8] It is now a simple consequence of [12], Theorem 1, to establish that social preference is ex-ante utilitarian with the common prior p.

$\square$

Example 2. *To see why we assume $|\Theta| \geq 2$, let us consider an example with $|N| = 3$, $\mathcal{T} = \{t_1, t_2\}$, and $\Pi_1 = \Pi_2 = 2^{\mathcal{T}}$, while $\Pi_3 = \{\varnothing, \mathcal{T}\}$. We will assume $X = \mathbb{R}$ and that each agent is risk neutral, while $p_3(t_1) = p_3(t_2) = \frac{1}{2}$. Now consider the social welfare function $U_0(f) = ((u_1(t_1) + u_2(t_1))^3 + u_3(t_1)) + ((u_1(t_2) + u_2(t_2))^3 + u_3(t_2))$. It is easily verified that this social welfare function satisfies the properties. It is also easily verified it is not of a utilitarian form (there is; however, obviously a common prior).*

We do remark; however, that the assumption $|\Theta| \geq 2$ is not necessary; but merely sufficient. For example, if we have two types and two agents, then the consequences of Theorem 1 continue to hold. However; given that all uncertainty here about preferences of agents involves beliefs; it is not clear what interpretation to give to a type space which does not build on uncertainty over objective states.

3 Conclusion

We provide a result extending Zuber [16]'s results to situations in which there is uncertainty about the preferences of other agents. In such an environment, the common knowledge of Pareto property is necessarily very weak. All the same, when imposed jointly with monotonicity, it generically results in an impossibility result. The only

[8] The separation result is originally due to Morris [11]. A particularly simple proof of this result is due to Samet [13]. It states that there is no common prior if and only if there exist, for all $i \in N$, $x_i \in \mathbb{R}^{\mathcal{T} \times \Theta}$ for which $\sum_i x_i = 0$ and for all $t \in \mathcal{T}$ and all $i \in N$, $E_{p_i}[x_i|\pi_i(t)] > 0$. Now let $g \in \mathcal{F}$ be any act for which for all $i \in N$ and all (t, θ), $u_i(g(t, \theta)) = 0$, and $f \in \mathcal{F}$ be any act for which $u_i(f(t, \theta)) = (1/\lambda_i)x_i(t, \theta)$. Consequently, $f \sim_0 g$ and it is commonly known that for all $i \in N$, $f \succ_i^t g$. This is a contradiction to common Pareto, so there must exist a common prior.

exception is when there is a common prior and when aggregation is utilitarian. This provides further argument for studying social welfare functions under uncertainty which depart from the monotonicity criterion.

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